



Laboratory analog and numerical study of groundwater flow and solute transport in a karst aquifer with conduit and matrix domains

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ABSTRACT

New mathematical and laboratory methods have been developed for simulating groundwater flow and solute transport in karst aquifers having conduits imbedded in a porous medium, such as limestone. The Stokes equations are used to model the flow in the conduits and the Darcy equation is used for the flow in the matrix. The Beavers–Joseph interface boundary conditions are adopted to describe the flow exchange at the interface boundary between the two domains. A laboratory analog is used to simulate the conduit and matrix domains of a karst aquifer. The conduit domain is located at the bottom of the transparent plexiglas laboratory analog and glass beads occupy the remaining space to represent the matrix domain. Water flows into and out of the two domains separately and each has its own supply and outflow reservoirs. Water and solute are exchanged through an interface between the two domains. Pressure transducers located within the matrix and conduit domains of the analog provide data that is processed and stored in digital format. Dye tracing experiments are recorded using time-lapse imaging. The data and images produced are analyzed by a spatial analysis program. The experiments provide not only hydraulic head distribution but also capture solute front images and mass exchange measurements between the conduit and matrix domains. In the experiment, we measure and record pressures, and quantify flow rates and solute transport. The results present a plausible argument that laboratory analogs can characterize groundwater water flow, solute transport, and mass exchange between the conduit and matrix domains in a karst aquifer. The analog validates the predictions of a numerical model and demonstrates the need of laboratory analogs to provide verification of proposed theories and the calibration of mathematical models.

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1. Introduction

Karst aquifers are valuable and essential resources and are extremely susceptible to contamination due to rapid transport processes and limited chemical filtering capacity that normally slows the spread of solutes in nonkarstic aquifers (Taylor and Greene, 2001; Matusick and Zanbergen, 2007; Kuniansky, 2008). Quantitative understanding of karst hydrologic functions is integral to managing water resources and developing protection or remediation strategies (Kincaid, 2004). However, traditional methods of aquifer characterization and testing,

based on Darcian approaches, provide misleading or inadequate quantitative data when applied to karst settings. This difficulty is partly a problem of scale (volume of aquifer tested), and partly a problem of the complex nature of the typical karst aquifer system.

Early approaches to studying karst concentrated on describing geomorphic features and their hydrologic functions, or understanding singular elements of karst flow dynamics, such as spring discharge, or hydraulic properties of solutional conduits. However, proper understanding of karst aquifers requires a systems approach in which the hydrologic function of each primary component–vadose zone, epikarst, and conduit network is considered separately and as an integrated part of the whole system. The major difficulty

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facing the hydrologist is that karst aquifers typically exhibit dual groundwater flow regimes, that is, fast (conduit-dominated) flow and slow (diffuse) flow. In selecting investigative techniques to characterize properties of a karst aquifer, it is therefore important to determine how the data obtained by a particular test method are influenced by the fast-flow regime, slow-flow regime, or both.

A karst aquifer typically has significant secondary porosity manifested by large interconnected cavernous conduits and caves or underground drainage channels (e.g., Ritter et al., 2002; Scanlon et al., 2003). Clearly, two pore systems exist in the aquifer, the conduits and caves as one system, and the surrounding porous limestone as another. Groundwater flow in the matrix is slow and the flow can be described by Darcy's law. However, the flow in the conduits and caves is fast and the flow state may be turbulent (Kincaid, 2004), so Darcy's law is obviously not satisfied in describing this kind of flow. Therefore, using Darcy-based modeling software, like MODFLOW (Harbaugh, 2005) and MT3D (Zheng and Wang, 1999) to simulate groundwater flow and solute transport in a karst aquifer is generally not successful; the predicted results deviate from field observations.

It has been observed that water and solute can exchange between conduits and porous matrix according to the hydraulic and chemical gradients between the two systems (White, 2002) and the residence time of a solute in limestone matrix is much longer than in the conduit. Katz et al. (1998) investigated the change in the geochemistry of groundwater caused by a major recharge pulse from the Little River, a sinking stream in North Florida. They observed that during high-flow condition, the chemistry of water in some of the monitoring wells changed, reflecting the entry of surface water from sinking rivers, and the proportion of river water that mixed with groundwater ranged from 0.10 to 0.67, based on a binary mixing model. Undoubtedly, their study shows the possibility of pollutants entering the limestone matrix via sinking streams is high, at least near the sinkholes. Chen (1993) investigated sinkholes as contamination pathways in Brandon karst terrain of central-west Florida. He found that sinkholes are predominantly local pathways for the introduction of contaminants and nitrogen compounds are dispersed along the major regional conduit, while the contamination from the surface application of agricultural chemicals and human wastes via infiltration is small because of a low permeability confining unit (Li et al., 2008). Kincaid (2004) found that there was significant tracer loss for during tracer tests on the Wakulla Springs near Tallahassee, Florida. The lost tracers may be attributed to retention within the ambient limestone matrix along the conduit. In contrast to observations that conduit-borne contaminants migrate into the groundwater within the limestone matrix, the reverse phenomena is that the sequestered contaminants are slowly released into the conduit, affecting the water quality of the conduit and associated springs over a prolonged period of time. Without this release, the breakthrough curve at the spring should end without backward skewness or extended tailing that often occurs in field monitor and observation. A desired model that can explain, detect and predict these phenomena requires incorporation of interaction between conduit water and matrix water, i.e., retention and releasing into the model (e.g., Geyer et al., 2007).

As shown in Fig. 1, during a high precipitation season, the water pressure or head in conduits are larger than in ambient limestone matrix, conduit-borne contaminant can be driven with flow into the surrounding matrix. While the incorporated contaminant will be slowly released back into the conduits and eventually produce severe skewness or a long tail in the breakthrough curve at the discharging spring during a low-flow season (Li et al., 2008). This retention and release phenomena produce an environmental problem in that the sequestered contaminant may influence the quality of underground water sources for a long time and significantly decrease the water availability. Quantification of the water and solute exchange between conduits and matrix is indispensable for accurate modeling of flow and transport in the aquifer.

Water and chemical exchanges between two porous medium systems within an aquifer are also considered in a fractured medium, fractures and matrix, where the water flow and solute transport are often described by a dual-porosity/permeability model (Gerke and van Genuchten, 1993a,b; Hu et al., 2002), in which interaction between fractures and matrix is modeled by an empirical solute-exchange coefficient, while the dynamic details of flow exchange between fractures and matrix is often ignored, which is reasonable if the pressure loss in fractures is very small and the permeability of the matrix is very low so that advection in the matrix is negligible. Even with considerable success in simulating fracture porous matrix, a dual-porosity model is not applicable to highly heterogeneous and anisotropic karst aquifers wherein a limited number of conduits cannot be adequately simulated by the continuum approach. In many karst regions, like Florida, groundwater and surface water are connected through sinkholes and springs. The variations of surface water and precipitation will lead to substantial pressure variation within conduits, which results in strong interaction between conduit water and water in the porous matrix. Therefore, the applicability of dual-porosity models without this water interaction is limited. The discharging pore-water flow and released contaminants should be incorporated to achieve a better description and prediction of groundwater flow and contaminant migration within the aquifer.

Recently, the dual-porosity model has been modified to simulate groundwater flow and solute transport in a karst aquifer (e.g., White, 2002; Liedl et al., 2003; Worthington, 2003; Shoemaker et al., 2008). Groundwater flow in matrix and/or fissured systems is described by a continuum approach and Darcy law. Flow in conduit system is represented by cylindrical tubes and governed by Kirchhoff's rule. Exchange of groundwater between the fissured/matrix and conduit system is modeled by a linear steady-state exchange term (Barenblatt et al., 1960), i.e., the flow rate is assumed to be proportional to the head difference between the flow systems and the exchange coefficient. This approach has greatly advanced modeling groundwater flow and solute transport in a karst aquifer with dual or triple flow systems, but there are also many limitations for the application of this method. First, the discharge in a conduit is assumed to be linearly proportional to the head gradient along the conduit, which may not be suitable if the flow is very fast. Second, the exchange rate can only be obtained by inverse calculation,

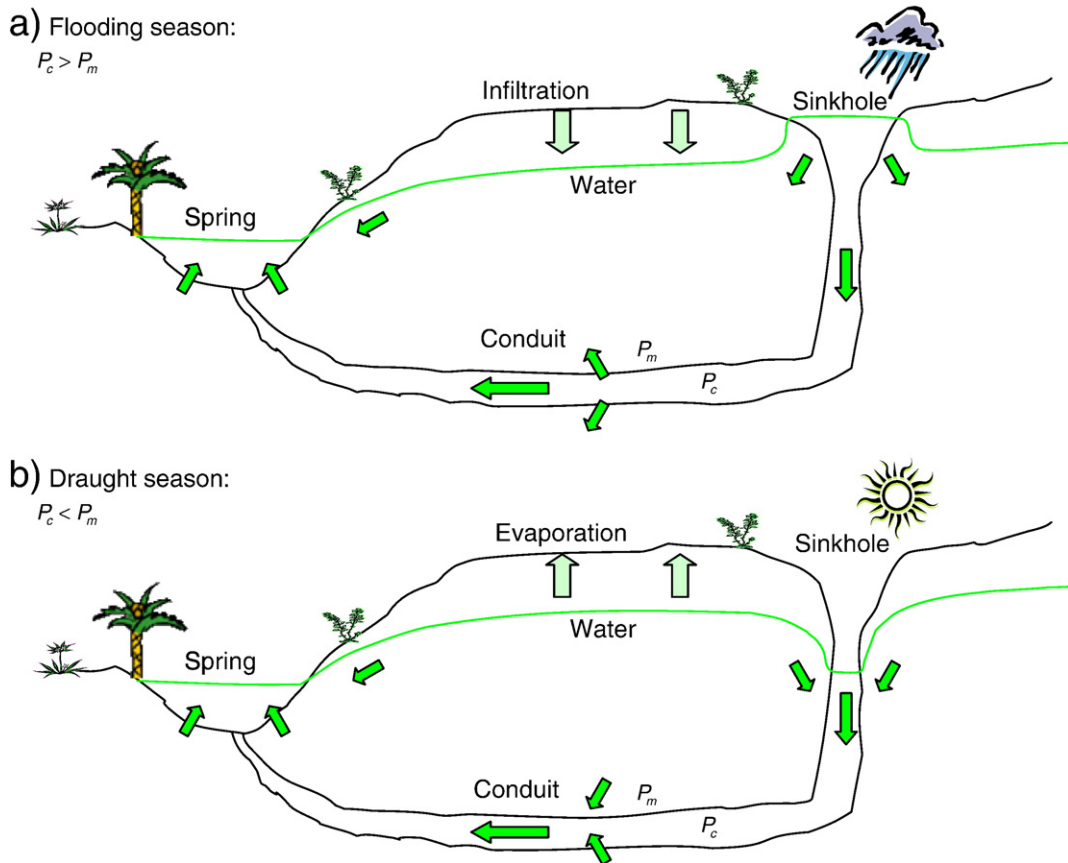


Fig. 1. The schematic water and contaminant interaction between conduit and limestone matrix with different hydrogeologic seasons. a) in a flooding season, the higher conduit water pressure drives a radial-flow which carries contaminants into limestone matrix; b) in a draught season, the higher water pressure in the matrix pushes water into the conduit and releases the contaminant back. , are the water pressure in the conduit and limestone matrix, respectively. From Li (2004).

which greatly limits its application for prediction in a large area with various conduits in different geological media. Third, the flow velocity and solute concentration are assumed to be uniformly distributed in the cross-section perpendicular to the conduit flow direction, which is generally not true.

Since the last decade, there have been a number of mathematical studies that use the Stokes or Navier–Stokes models to simulate the flow in the conduits (e.g., Burman and Hansbo, 2005, 2007; Cao et al., submitted for publication). These studies focus on the interaction between the free flow in the conduits and the confined flow in the matrices. Both theoretical studies and numerical simulations indicate that such an approach has the potential of improving the accuracy of the simulation results over those obtained by dual-porosity models. However, the coupling of Stokes or Navier–Stokes systems with a Darcy system to study groundwater flow in a karst aquifer with a proper modeling of the hydraulic boundary between the conduit and matrix is still a relatively new area of inquiry.

In this study, we use a laboratory analog experiment to simulate groundwater flow and solute transport in a karst aquifer with one conduit buried in matrix. The experiment mainly focuses on the water and solute exchanges between the matrix and conduit. The experimental results will serve as the benchmark for numerical study. Based on a dual-regional conceptual model, we develop a numerical model to simulate

groundwater flow and solute transport in a karst aquifer. The karst aquifer is divided into two regions, limestone matrix and conduit. The groundwater flow in matrix is assumed to satisfy Darcy's law, but the flow in conduit is described by Navier–Stokes equation. The groundwater and solute exchanges between the two domains are treated as an interface action, no first-order mass diffusion model is assumed here. The numerical method is used to study the sandbox experiment results.

2. Conceptual and mathematical modeling formulation

Fig. 2 provides a two-dimensional sketch of the conceptual model we propose. We assume that a karst aquifer system consists of conduit and matrix regions Ω_c and Ω_m , respectively. The boundary of the conceptual karst aquifer is divided in the sinkhole Γ_{si} , the spring Γ_{sp} , the ground Γ_g , and the artificial boundary Γ_0 created when the domain is truncated. The interface boundary between the conduit and matrix is denoted by Γ_{cm} . The free flow is confined in the conduit, which connects the sinkhole and the spring. Surrounding the conduit is a porous medium such as limestone. The porous medium as a whole is regarded as the matrix holding water. We use the subscripts m and c to indicate the domain of definition of the model variables.

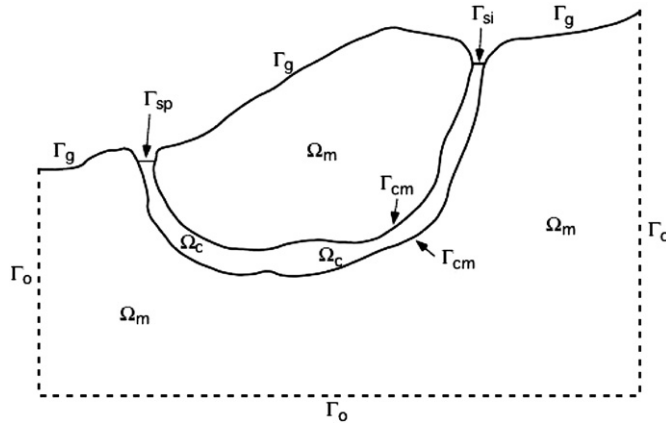


Fig. 2. Conceptual model of a karst aquifer having a conduit embedded in a matrix.

In the matrix Ω_m , the flow is governed by the Darcy system (Bear, 1972)

$$\left. \begin{aligned} S \frac{\partial h_m}{\partial t} + \nabla \cdot q_m &= f_m \\ q_m &= -K \nabla h_m \\ h_m(x, t = 0) &= h_0 \end{aligned} \right\} \quad \text{in } \Omega_m, \quad (1)$$

where q_m denotes the specific discharge (that can be expressed as $q_m = n v_m$, where v_m denotes the seepage velocity and n the effective porosity), S the storage coefficient, K the hydraulic conductivity tensor, and f_m represents sink/source term. The hydraulic head h_m is defined by $h_m = z + \frac{p_m}{\rho g}$, where p_m denotes the pressure, ρ the water density, g the gravity acceleration, and z the position head.

By substituting the second equation in (1) into the first one, we obtain the equation that governs the change of the hydraulic head:

$$S \frac{\partial h_m}{\partial t} + \nabla \cdot (-K \nabla h_m) = f_m \quad \text{in } \Omega_m \quad (2)$$

The boundary conditions for (2) are the Dirichlet boundary condition $h_m = H(\mathbf{x})$ along Γ_g , where $H(\mathbf{x})$ is known from measurements, and the homogeneous Neumann boundary condition $(K \nabla h_m) \cdot \mathbf{n} = 0$ along Γ_o that represents a no-flow boundary condition at the artificial boundary of the aquifer.

In the conduit Ω_c , the Stokes equations govern flow:

$$\left. \begin{aligned} \frac{\partial v_c}{\partial t} - \nabla \cdot \mathbf{T} &= f_c \\ \nabla \cdot v_c &= 0 \\ v_c(x, t = 0) &= v_0 \end{aligned} \right\} \quad \text{in } \Omega_c \quad (3)$$

where v_c denotes the fluid velocity, $T(v, p) = -pI + 2\nu D(v)$ the stress tensor, p_c the kinetic fluid pressure, $D(v) = \frac{1}{2}(\nabla v + (\nabla v)^T)$ the deformation tensor, ν the kinetic viscosity of the fluid, and f_c a general body forcing term.

At the sinkhole and the spring, we apply nonhomogeneous Dirichlet boundary conditions that specify the inflow and outflow velocities, respectively. Specifically, we set

$$v_c \times n = 0 \text{ and } v_c \cdot n = \gamma_{si}(t) \eta_{si}(x) = f_{si} \text{ on } \Gamma_{si} \quad (4a)$$

$$v_c \times n = 0 \text{ and } v_c \cdot n = \gamma_{sp}(t) \eta_{sp}(x) = f_{sp} \text{ on } \Gamma_{sp} \quad (4b)$$

where γ_{si} , γ_{sp} , η_{si} and η_{sp} are given functions defined at the spring and sinkhole. These boundary conditions are determined from measurements.

Along Γ_{cm} , the interface boundary between the matrix and conduit, we apply the Beavers–Joseph conditions (Beavers and Joseph, 1967)

$$\left. \begin{aligned} v_c \cdot n_{cm} &= v_m \cdot n_{cm} \\ -n_{cm}^T T(v_c, p) n_{cm} &= g(h_m - z) \\ -\tau^T (v_c - p) n_{cm} &= \frac{\alpha \gamma \sqrt{d}}{\sqrt{\text{trace}(k)}} \tau^T (v_c - v_m) \end{aligned} \right\} \quad \text{on } \Gamma_{cm} \quad (5)$$

where τ denotes a local orthonormal basis for the plane tangent to Γ_{cm} and α is a constant parameter. The first condition states that the water mass flow is conserved across Γ_{cm} , the second that the pressures on the both sides of Γ_{cm} are the same, and the third equation relates the tangential stress on the conduit side of Γ_{cm} to the jump in the tangential velocity across Γ_{cm} . Justifications for the Beavers–Joseph interface conditions can be found in Beavers and Joseph (1967).

The well-posedness of the coupled Stokes–Darcy model with the Beavers–Joseph interface boundary conditions was studied in Cao et al. (submitted for publication). In the steady-state case, the well-posedness is established under the assumption of small parameter α in the Beavers–Joseph interface condition. In the time-dependent case, well-posedness is established, under the assumption that the matrix media is isotropic, for general α via an appropriate time discretization of the problem and a novel scaling of the system. The convergence of finite element discretizations of the coupled system systems and error estimates for the finite element approximations were obtained in Cao et al. (in press).

Computational experiments verified that the code implementing the finite element methods produced approximations that achieved the theoretical convergence rates. In this study, we developed a numerical code to validate the coupled Stokes–Darcy model with the Beavers–Joseph interface boundary conditions, i.e., to demonstrate, by comparing to experimental results, that it accurately models a simplified karst aquifer system.

The velocity determined from the coupled Stokes–Darcy system is used in the governing equation for the tracer evolution in the matrix:

$$\frac{\partial C_m}{\partial t} + v_m \cdot \nabla C_m - D \nabla^2 C_m = 0 \quad (7)$$

where C_m denotes the solute concentration in matrix and D is dispersion coefficient. Ideally, one should couple the tracer density in the matrix and the conduit. However, assuming that the flow in the channel moves significantly faster than that in the matrix, we simply impose a Dirichlet boundary condition along the interface Γ_{cm} for the tracer density in the matrix and homogeneous Neumann boundary condition elsewhere on the matrix boundary.

3. Laboratory analog and experiment procedure

Glass beads (30–40 US Sieve or 590–420 μm) were used for the matrix material. A permeameter was fabricated to determine the conductivity of the glass beads used in this experiment. Water was allowed to flow into the unit and trapped air was removed using a venture vacuum system. Vinyl tubing was connected to a fitting located on top of the analog while the other end of the tubing was connected to a venturi on a laboratory faucet. With the faucet on, a slight vacuum was generated within the analog. Water flowed in from the supply reservoir to replace water and air removed by the venturi. With the outlet tube at a fixed elevation and a constant head water supply to the permeameter, five measurements of outflow were taken using a graduated cylinder and a stop watch. The Darcy equation is used to determine the conductivity of the glass bead, which is $0.0619 \pm 0.0005 \text{ cm/s}$. The porosity of the beads was calculated to be 0.394 through the equation $n = (V_v / V_t)$, where n = porosity (Dimensionless), V_v = the volume of void space (L^3), and V_t = the total volume (L^3) (Hornberger et al., 1998). To determine the porosity of the glass beads, a clean and dry 100 ml beaker was filled with glass beads in 20 ml increments and tapped on the side and bottom of the beaker to produce closest packing. Using titration with a water temperature of 21.4 °C, the device was filled to 100.0 ml. Water was added to the beaker until it filled the beaker to the level of the glass beads. The added water volume was 31.5 ml. The porosity of the glass beads was approximately 39% of the total volume.

A diagram of the principle components of the laboratory analog is shown in Fig. 3 and the equipment in the laboratory is shown in Fig. 4. The laboratory analog is constructed of transparent, acrylic plexiglas to allow digital imaging of the dye tracing experiments and provide support for instrumentation and the matrix of glass beads. The analog has two domains. The conduit domain occupies the bottom of the model and the interior measures 57.8 cm $\times$ 2 cm $\times$ 2 cm. The matrix domain occupies the rest of the space and has an inflow

and outflow reservoir on either side the porous medium section. The interior of the matrix including the inflow and outflow reservoirs measures 57.8 cm $\times$ 24.1 cm $\times$ 2 cm. The interior of the porous medium portion of the matrix measures 47.8 cm $\times$ 24.1 cm $\times$ 2 cm. The inflow and outflow reservoirs measure 5 cm $\times$ 24.1 cm $\times$ 2 cm. The reservoirs and matrix material are separated by a stainless steel screen. The conduit is separated from the matrix domain by stainless steel screen and plexiglas. The type 304 stainless steel screen used in the laboratory analog to separate the two domains has the following properties: mesh size 40 $\times$ 40, square openings of 406 μm , a wire diameter of 241.3 μm and a 38.4% of open area.

There are 28 ports available for measurements of water flow rates and hydraulic heads. The conduit has one inflow and one outflow ports as well as five ports for pressure transducers, which are used for measurements of hydraulic heads. The matrix domain has two inflow and two outflow ports. The matrix inflow and outflow reservoirs have one transducer port each and the main body of the matrix has fifteen transducer ports. Each domain has its own constant head water supply with the same components as depicted in Fig. 3. Tap water from a laboratory faucet is routed through an inline water filter and then to the flow meters at the inflow entrances to the laboratory analog. From the filter, water flows into a module that contains a deaeration membrane. The water is then routed through a float-style fill valve that maintains constant head in each supply reservoir container. For imaging purposes, dye can be added to a supply reservoir during an experiment.

The flow into each domain is measured by flow meters. The conduit flow meter, Omega model FLR 1010, has a flow range of 1.0 ml/s to 16.7 ml/s. The matrix flow meter, Omega model FLR 1008, has a flow range of 0.33 ml/s to 3.33 ml/s. Both meters have an accuracy of $\pm 3\%$ of full scale and repeatability of $\pm 0.2\%$ of full scale. The flow meters are located downstream of the control valve just before the inflow ports of each domain.

Water exiting the conduit flow meter is carried by vinyl tubing to the conduit domain port. Water exiting the conduit may follow one of two paths depending upon the experimental setup. Water may exit the conduit via the conduit outflow port or enters the matrix domain through the stainless steel screen dividing the two domains and exit via the matrix reservoir outflow ports.

Water exiting the matrix flow meter is carried by vinyl tubing to the matrix domain inlet ports. Water exiting the matrix domain may follow one of three paths depending on the experimental setup. Water may travel through the matrix and enter the conduit through the screen dividing the two domains and exit via the conduit outflow port. It may also reenter the matrix and travel through to the matrix reservoir. Water exits the domain through two ports and into vinyl tubes before entering a y-shaped fitting that joins the two flows into one tube. Water may also enter the matrix and exit from the matrix without entering the conduit domain.

The analog has a total of 22 ports available for measuring head. The Freescale Semiconductor transducer model MPX5010GP is used in this study, which is temperature compensated with a pressure range of 0–10 kPa and a 0.2–4.7 V output. Calibrated accuracy was measured to be 0.10%.

Power for the flow meters was supplied by an AC to DC, 12 V transformer. Voltage signals produced by the flow meters

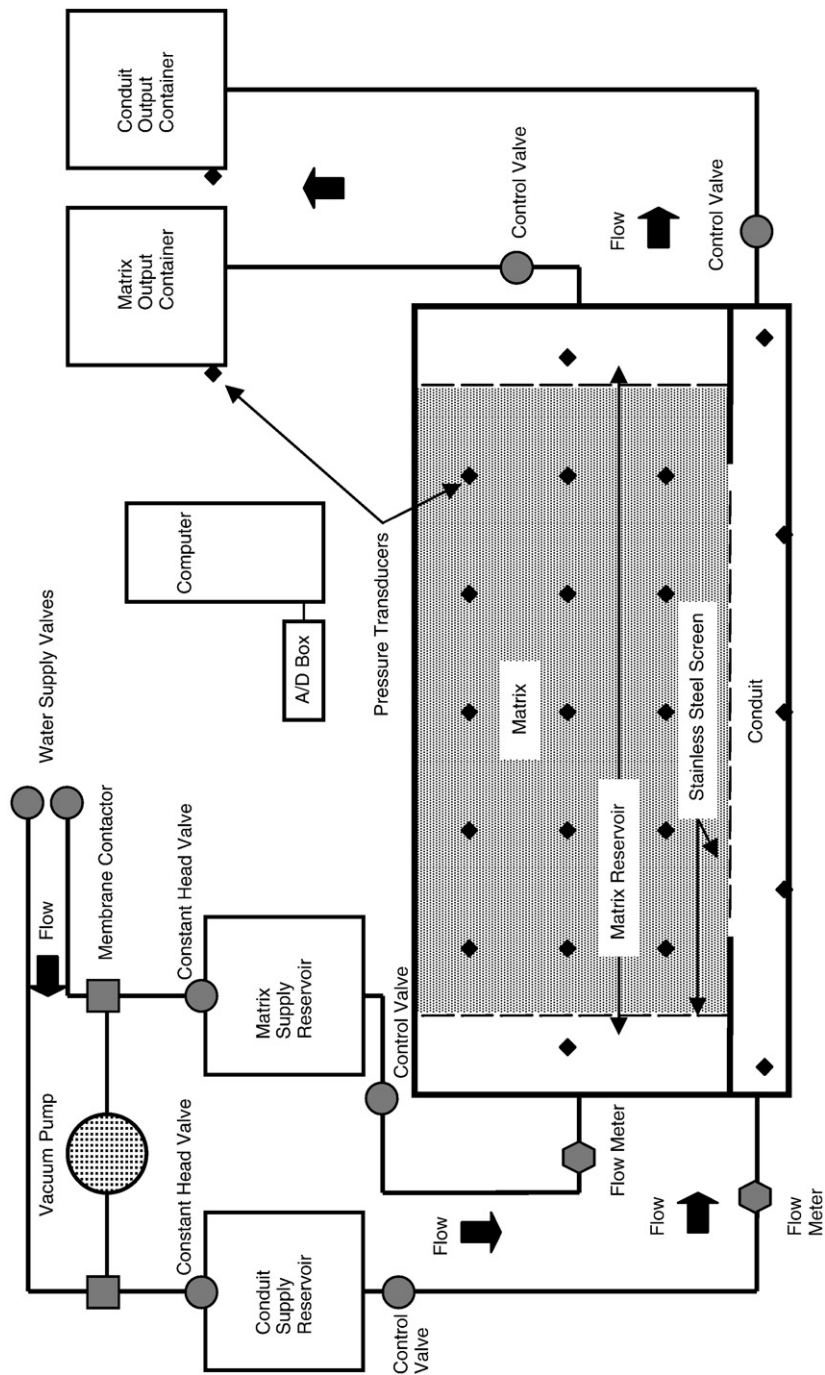


Fig. 3. Diagram of the principle components of the laboratory analog.



Fig. 4. Photograph of the laboratory setup.

and transducers were carried to the A/D box by shielded, instrumentation and computer cable. The digitized data were analyzed using National Instruments LabVIEW7.1 (Laboratory Virtual Instrumentation Engineering Workbench).

Data for both pressure and flow (1000 samples/s) were averaged and the results were recorded by LabVIEW in the form of a spreadsheet. The GBTimelapse program was initiated and captured images every 10 s for the duration of the experiment. All inlet and outlet control valves were opened and the experiment was monitored. Screen shots of the front panel of LabVIEW were recorded at the beginning, middle, and the end of the experiment for visual snapshots of hydraulic heads and flow rates during the experiment. After the experiment was started, four screen shots were captured at 10 s intervals. Midway through the experiment, eight screen shots were taken at 20 s intervals and four screen shots were recorded at the end of the experiment at 10 s intervals. The average and standard deviation for each transducer and flow meter were calculated. The standard deviation for the conduit flow meter was ± 0.0029 ml/s and ± 0.0012 ml/s for the matrix flow meter. The standard deviation for transducers 0–21 ranged from ± 0.03 cm to ± 0.56 cm of head. Total head and flow rates from the screen shot captured at $t=20$ s

midway through the experiment, were used in conjunction with transducer coordinates to generate actual total head pressure information on an image. Total head distribution was contoured to allow a graphical visualization of pressures across the laboratory analog. With this information, flow paths were developed. A starting point was selected on a contour interval. The minimum distance to the next contour interval was determined and recorded and the procedure continued. When the process was complete, a flow path was established.

The solute transport process in the system was simulated through a dye experiment. Imaging of the dye was done using a digital camera. All of the camera's operation including settings and image capture are controlled by GBTimelapse program from Granite Bay Software and is a Microsoft Windows application for the capture of time-lapse images. The dye was added as a step input. Physically, we added a large dose of dye into the conduit water supply budget. The dye diluted as some exited the budget. Since the dye experiments were conducted in a short period, generally less than 10 min, no additional dye was added. Since the dye water has a slightly greater density and viscosity than the fresh water, the experiment was set up with the head in the conduit being greater than the head in the matrix and water

Table 1

Experimental setup for experimental configuration 4.

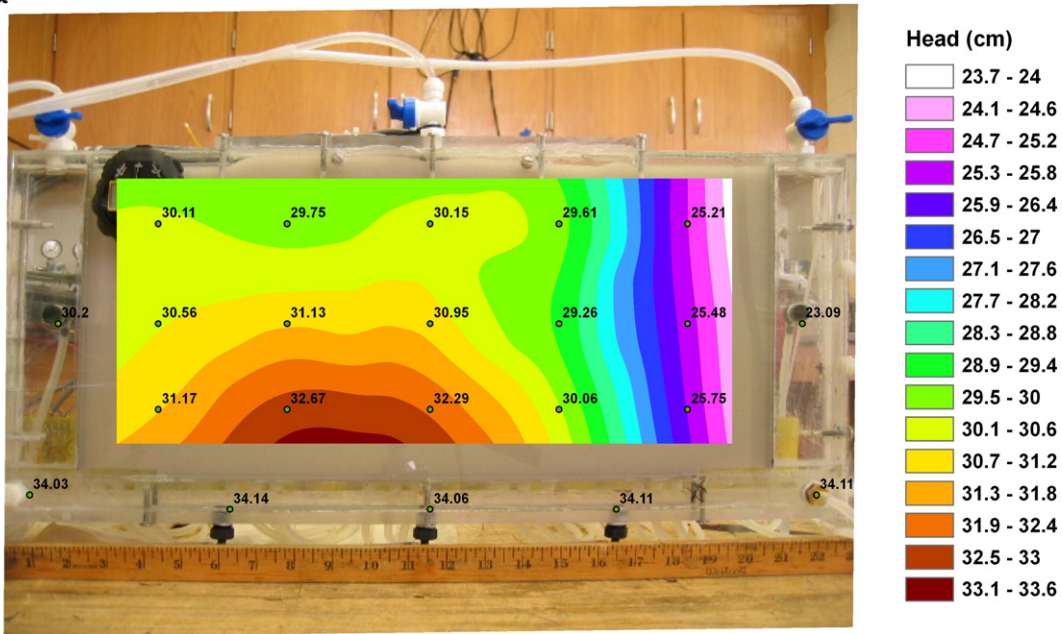
Date	6/20/2008
Configuration	Matrix head > conduit head
Experiment	4
Conduit head	64.2 cm
Conduit supply temperature	26.1 °C
Matrix head	61.6 cm
Matrix supply temperature	21.4 °C
Total time (s)	3239

Table 2

Summary of flow rates and volumes of experiment Number 4.

Total time (s) 3239	Conduit	Matrix	Porous media
Inflow volume (L)	11.37	1.39	
Inflow rate (ml/s)	3.48	0.43	
Outflow volume (L)	8.73	1.44	
Outflow rate (ml/s)	2.70	0.45	
Volume displaced in matrix (L)			0.39
Volume of leakage (L)			0.09

a



b

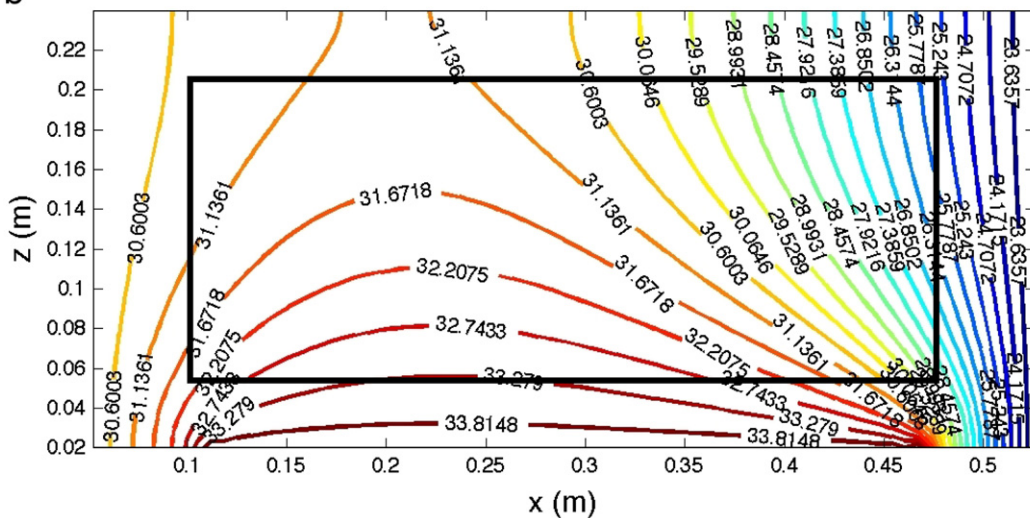
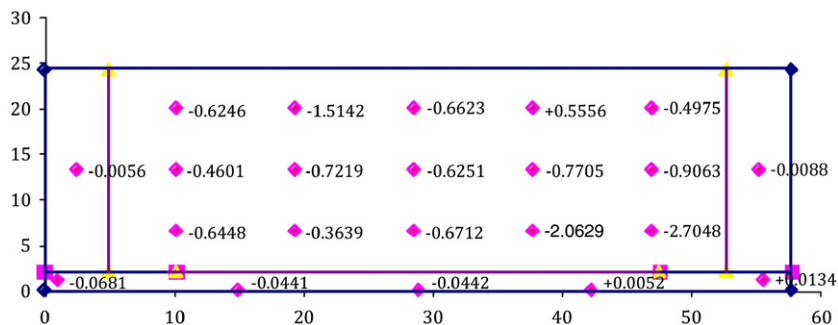


Fig. 5. Experimental (above) and simulated (below) head distribution in the matrix.

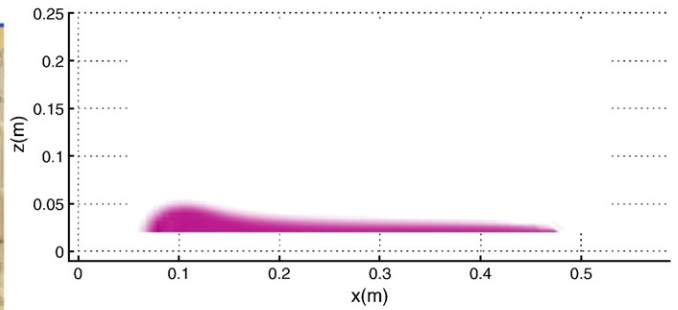
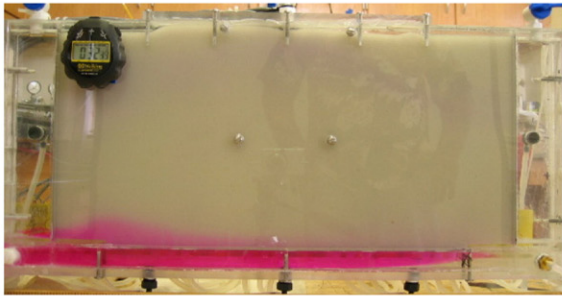
Fig. 6. Hydraulic head difference ($h_{lab} - h_{sim}$) (cm) between lab readings (h_{lab}) and simulation readings (h_{sim}) marked on each sensor location.

from the conduit is expected to flow up and into the matrix so that instability in the displacement front was minimized.

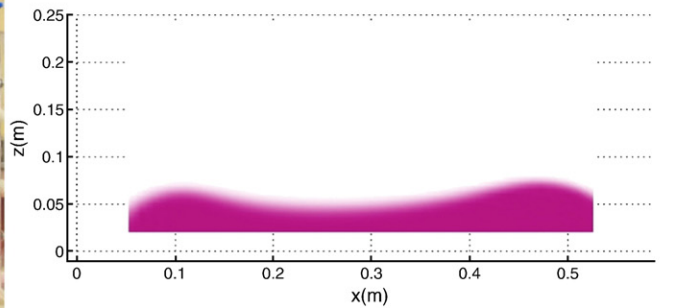
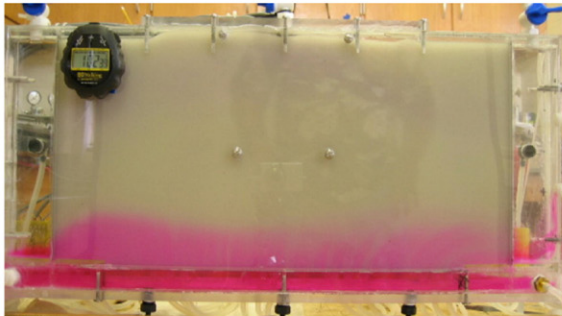
Of the 338 images generated during the course of this experiment, 12 were selected to be analyzed using the Spatial Analysis tool in ArcGIS9.2. The first image rectified was recorded 180 s after GBTimeLapse was initiated. The second

image used was recorded 175 s later as dye had passed into the matrix inflow reservoir and just began to enter the matrix. The remaining ten images were chosen at 300 s intervals to allow a clear progression of the dye front as it passed through the laboratory analog. Details of the selected run are reported in Table 1.

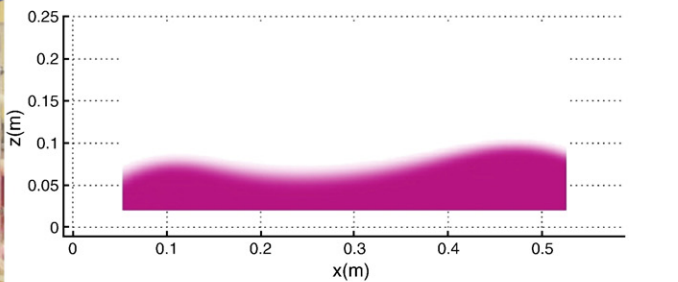
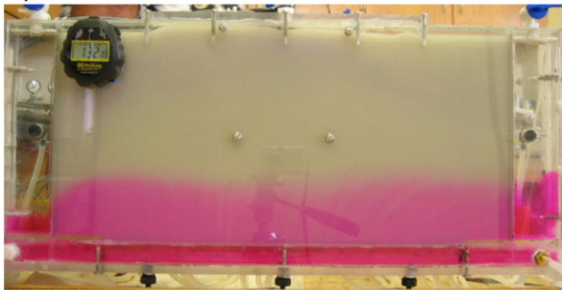
a) $t = 32\text{s}$



b) $t = 62\text{s}$



c) $t = 92\text{s}$



d) $t = 122\text{s}$

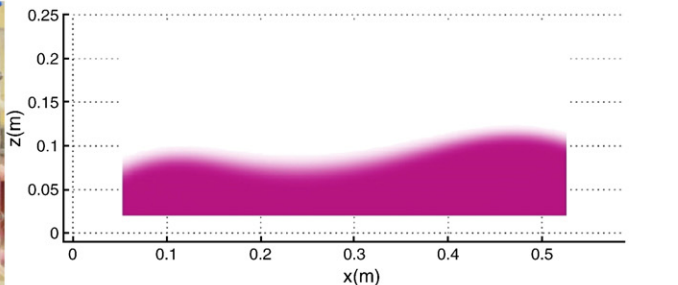
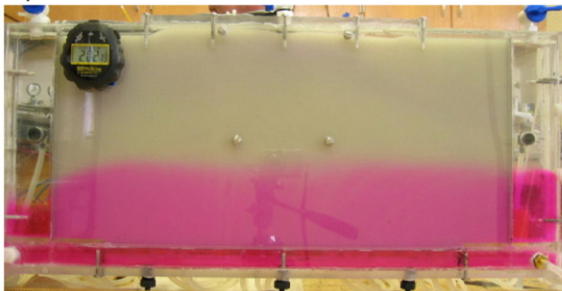


Fig. 7. Experimental and simulation results for the solute concentration distribution in the matrix at various time instants; top to bottom: $t = 32\text{ s}$, $t = 62\text{ s}$, $t = 92\text{ s}$, and $t = 122\text{ s}$.

Several similar experimental cases were conducted at different head conditions. A representative run was selected for detailed analysis. The experimental run time was approximately 54.0 min. To obtain tracer profiles a screen shot of the front panel of LabVIEW was recorded. Fluorescent red dye tracer was used in experiment to visually trace water flow and aid in determining velocities. The LabVIEW program was used to record and convert analog voltage signals to actual pressures, flow rates, and volumes.

4. Experimental and numerical study results

Table 1 gives the parameters for the experiment and Table 2 provides a summary of the experimental results. The total inflow and outflow are slightly unbalanced probably due to the measurement errors of the flow meters. Fig. 5 shows the experimental and simulated distributions of the hydraulic head in the matrix domain. For the experimental results, the hydraulic head data at the measurement points are interpolated to the areas between the data points using the software ArcGIS9.2 (2008). Therefore, there are no head values in the area not covered by the data points. For the simulation results, the simulation results look very smooth except two small areas around the two singular points at the end points of screen interface between the matrix and the conduit. Fig. 6 shows the head difference between the measurement and the simulation at each measurement point. From Figs. 5 and 6, one can see that in general, the simulated head values are slightly higher than the measurement results, but the head distribution patterns match very well. It should be pointed out that in most current pipe-flow models (Bauer et al., 2003; Birk et al., 2003; Liedl et al., 2003), the first-order mass exchange assumption is made and the mass exchange rate is difficult to determine. This assumption was not used in the

model developed in this study, so the mass exchange rate is not required.

Fig. 7 exhibits the experimental and simulated results for the dye distribution at four time instants. One can see from the figures that the results match very well at all time shots. At an early time, the dye in the conduit first flows into the matrix from the left end of the screen interface, but at a later time, more dye moves into the matrix from the right end of the interface. The dye distribution has like a broad “U” like shape. The dye movement in the matrix is consistent with the hydraulic head distribution. It is shown from Fig. 5 that the hydraulic gradient in the right side is larger than in the left, which in turn is larger than in the middle. Therefore, the dye moves faster at two ends than in the middle. It is also shown in Fig. 7 that the dye dispersion zone (the change in the dye concentration from 1 to 0) is narrow because the medium is made of roughly the same size beads and the beads are uniformly packed in the box.

The contoured hydraulic head from experimental results allow a graphical visualization of pressures across the laboratory analog. With this information, flow paths can be developed. In the upper portion of the laboratory analog, flow from the matrix inflow reservoir traveled to a low pressure area located near the top of the analog and exited to the matrix outflow reservoir.

From a digit image at a specific time, the dye front could be determined through recognized color change by naked eyes. Using eight of the nine digital images, individual dye fronts were established and shown in Fig. 8. Each dye front trace displays the time, in seconds when the dye front was at that position. There were areas where the dye became diffuse but was included in the tracing as it indicated flow to those regions. The dye also confirmed that there was some flow across the top of the analog. This is undoubtedly due to settling of the porous medium creating void spaces and causing an increase in conductivity.

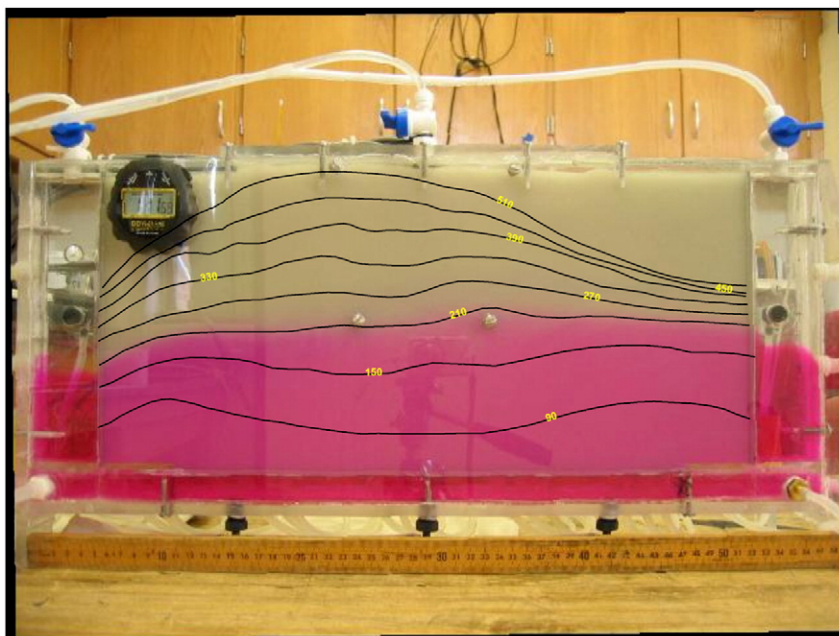


Fig. 8. Progression of the dye front over time. The time is displayed in seconds on each line superimposed on the image. Time interval between each line is 60 s.

5. Conclusions

We have developed a new modeling approach to simulate groundwater flow in a karst aquifer having conduit and matrix regions. The Darcy system is used to describe groundwater flow in the matrix, but Stokes equation is adopted to describe flows in conduits. The Beavers–Joseph interface conditions are applied at the interface between the two regions. The proposed model has previously been shown to be mathematically well posed and finite element approximation has been shown to converge. This model provides a more detailed description of the karst flow system than current pipe-flow models. Furthermore, the model does not need to assume that the water mass exchange between matrix and conduit satisfies the first-order rate equation; this assumption is required for pipe-flow model development and the mass exchange rate parameter in that model can hardly be directly measured.

We have also developed a new laboratory analog to physically simulate groundwater flow and solute transport in a karst aquifer with one straight conduit in a matrix. The experiments provide not only hydraulic head distribution but also capture solute front images and mass exchange measurements between the conduit and matrix domains. In the experiment, we measure and record pressures, and quantify flow rates and solute transport. This study proved the feasibility of using a laboratory analog to generate the data needed to better understand groundwater flow and solute transport between the conduit and matrix domains. Experimental data can also be used to verify and validate the mathematical and numerical models. The numerical simulation results for flow and solute transport match well with laboratory experimental results. Thus, the developed mathematical and numerical models are physically verified and validated by comparison to the experimental results for a set up that contains all the feature of karst aquifers expect for geometrical complexity.

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