Florida Department of Environmental Protection

Hydraulics 1: The Basic Principles of Hydraulics

Mariano Guardo PhD, PE, D.WRE

April 14, 2015





- Conservation of **Mass**:

  Continuity Equation

- Conservation of **Momentum**:

  Newton's 2^{nd} Law of Motion (forces and momentum)

- Conservation of **Energy**:

  Bernoulli Equation

The Conservation Principles

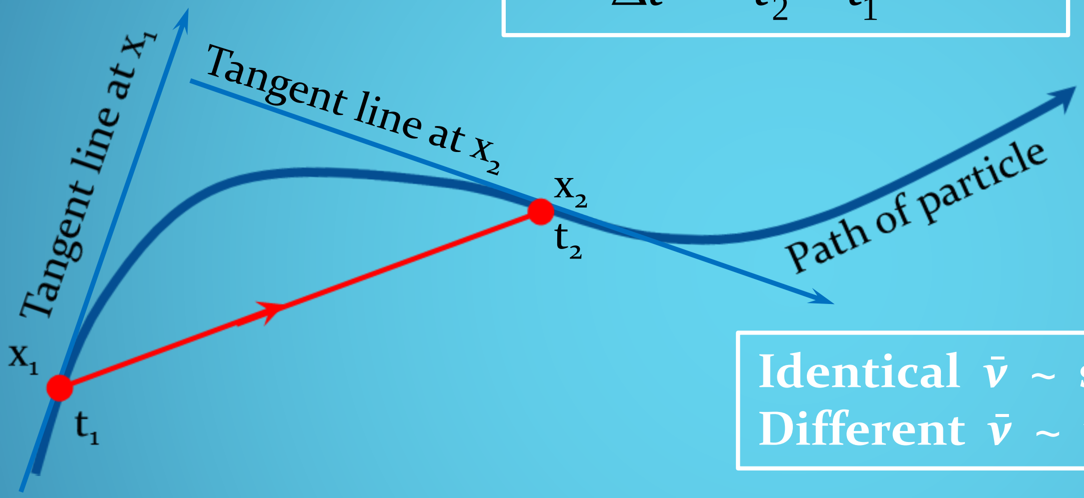
- **Flow Rate** or **Discharge**: Volume of water flowing through a certain cross-sectional area during a specified time period (incompressible fluids)

$$\boxed{Q = \frac{Volume}{Time} \qquad [1.1]}$$

Conservation of Mass

- **Flow Velocity**: Change of position of a water particle in the moving fluid during a certain time interval

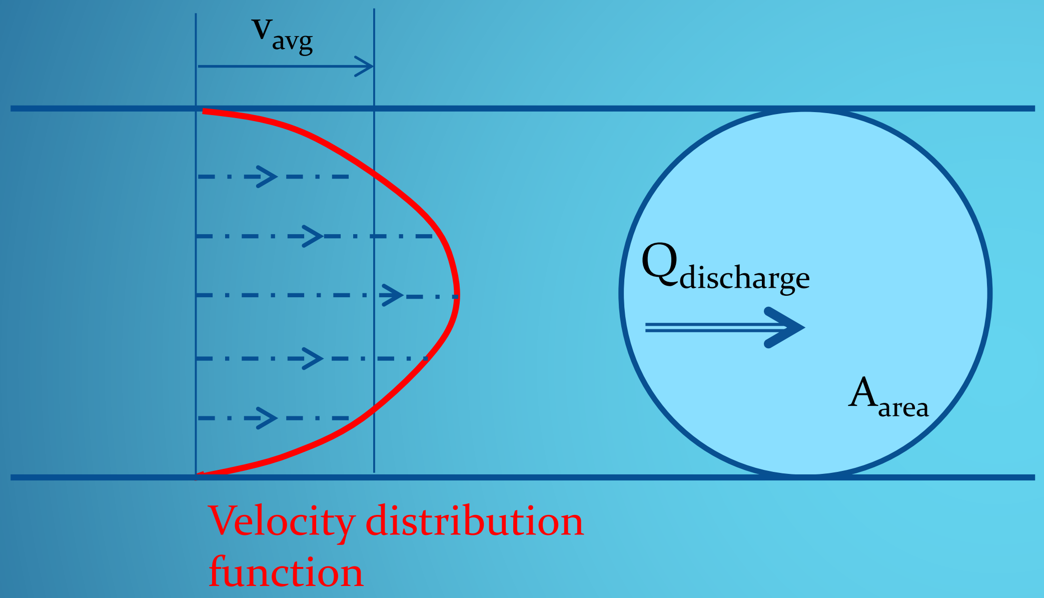
$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \quad [1.2]$$



Identical $\bar{v}$ ~ steady flow
Different $\bar{v}$ ~ unsteady flow

Conservation of Mass

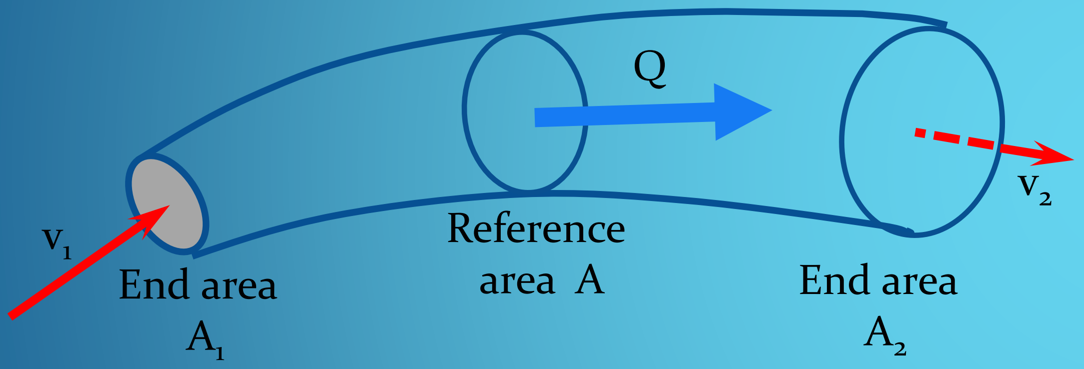
- **Discharge (Q) and Flow Velocity (v) Relation**: The flow velocity v changes spatially, but the average velocity can be estimated



$$v_{avg} = \frac{Q}{A} \quad [1.3]$$

Conservation of Mass

- **Continuity Equation**: For any hydraulic system, the discharge flowing in, the stored volume within, and the discharge flowing out must be balanced (because water is considered to be incompressible, mass or volume may be used interchangeably).



$$\gamma_1 v_1 A_1 = \gamma_2 v_2 A_2$$

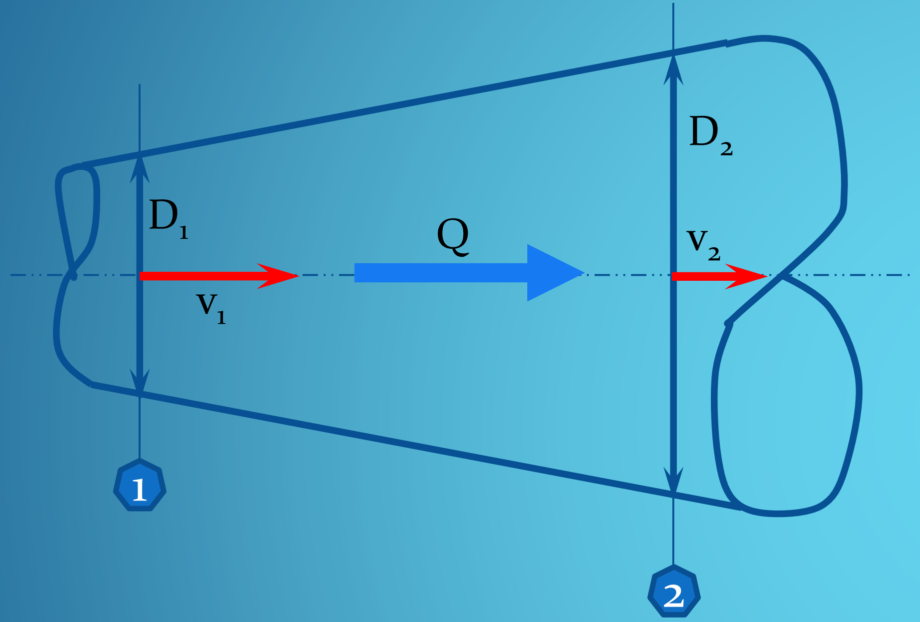
(in terms of the weight flux with units of lb/sec)

$$\gamma_{water} = 62.4 \text{ lb/ft}^3$$

$$\boxed{v_1 A_1 = v_2 A_2 = Q \quad [1.4]}$$

Conservation of Mass

- **Acceleration**: Rate of change in velocity. Velocity can change in place (spatial) as well as in time (temporal)



$D_1 < D_2$

$A_1 < A_2$

Continuity Equation

$Q = v_1 A_1 = v_2 A_2$

$v_1 > v_2$

$$\boxed{a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \quad [1.5]}$$

This yields negative acceleration (or deceleration)

Conservation of Mass

Conservation of Momentum

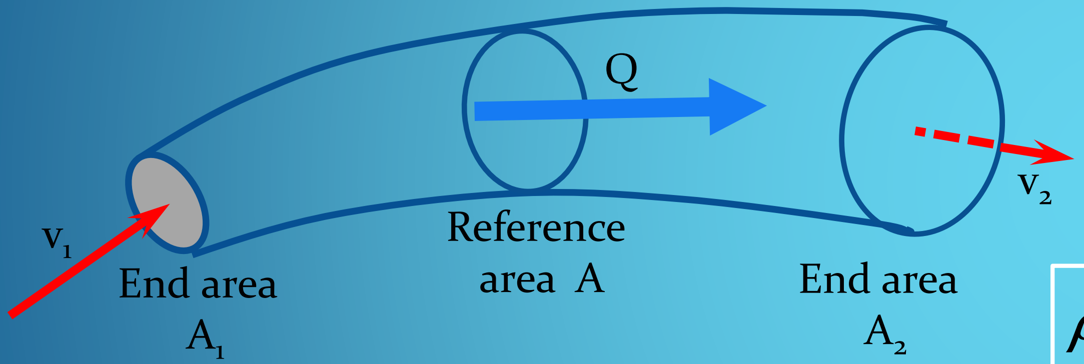
- By a similar analysis to that of conservation of mass, the equation of conservation of momentum along a streamtube (from Newton's) becomes,

$$F = \rho\,\beta\,Q\,(\Delta v/\Delta t) = (\Delta/\Delta t)\,\{\rho\,\beta\,Q\,v\}$$

(in terms of the momentum flux in lb)

$\rho_{water} = 1.94$ lb-sec^2/ft^4 (slugs/ft^3)

β = momentum coefficient ≈ 1.0



$$\rho_1\,Q\,v_1 = \rho_2\,Q\,v_2$$

$$\boxed{\rho_1 v_1^{\,2} A_1 = \rho_2 v_2^{\,2} A_2 \quad [1.6]}$$

- **Momentum Equation**: The previous equation indicates that the momentum is conserved only if the velocity remains constant through the streamtube

$$\boxed{F = \rho Q v = \rho v^2 A \quad [1.7]}$$

The momentum flux has the dimension of a force, and therefore is a vector, rather than a scalar. The equation may be also accordingly expressed in terms of the Cartesian components by using the velocity vectors

Conservation of Momentum

$$\Sigma F_x = \rho Q(v_{2x} - v_{1x}) \quad [1.8]$$

$$\Sigma F_y = \rho Q(v_{2y} - v_{1y}) \quad [1.9]$$

- Forces in the x- and y-direction generated by a moving fluid are a function of the pressure exerted in the fluid, and they are calculated as follows,

$$F = p\,A \quad [1.10]$$

where p is pressure (f/L^2) and A is the cross section of the flow (L^2). Note that when A is in ft^2 and p is in psi (lb/in^2), the latter has to be multiplied by 144 in^2/ft^2 to attain consistent units

Conservation of Momentum

- **Bernoulli Equation:** It represents a statement of the conservation of energy principle for fluids in motion. In fluid dynamics, an increment in flow velocity follows simultaneously with a decrease in pressure or a decrease in potential energy of the fluid. In every flow process, as in any physical "conservative" process, energy cannot be lost, although it may be converted into other forms (First law of Thermodynamics). For a hydraulic system, the sum of all energies is a constant. Energy is defined as the capacity for doing work (units of *ft-lb* of force per *lb* of mass) and in hydraulics, it is conveniently expressed as head (*ft*)

Conservation of Energy

- The Bernoulli Equation is defined as follows,

$$H = \alpha \frac{v^2}{2g} + z + \frac{p}{\gamma} \quad [1.11]$$

H = Total Head
α = Kinetic Energy Coefficient (≈1.0)
g = Acceleration of Gravity (32.17 ft/sec²)
γ = Specific Weight (62.4 lb/ft³)

Kinetic Energy = α v²/2g
Potential (elevation) Energy = z
Potential (pressure) Energy = p/γ

- As the flow remains in motion, there is an additional component of energy that is converted into heat. This conversion is produced from the shearing stresses resisting

Conservation of Energy

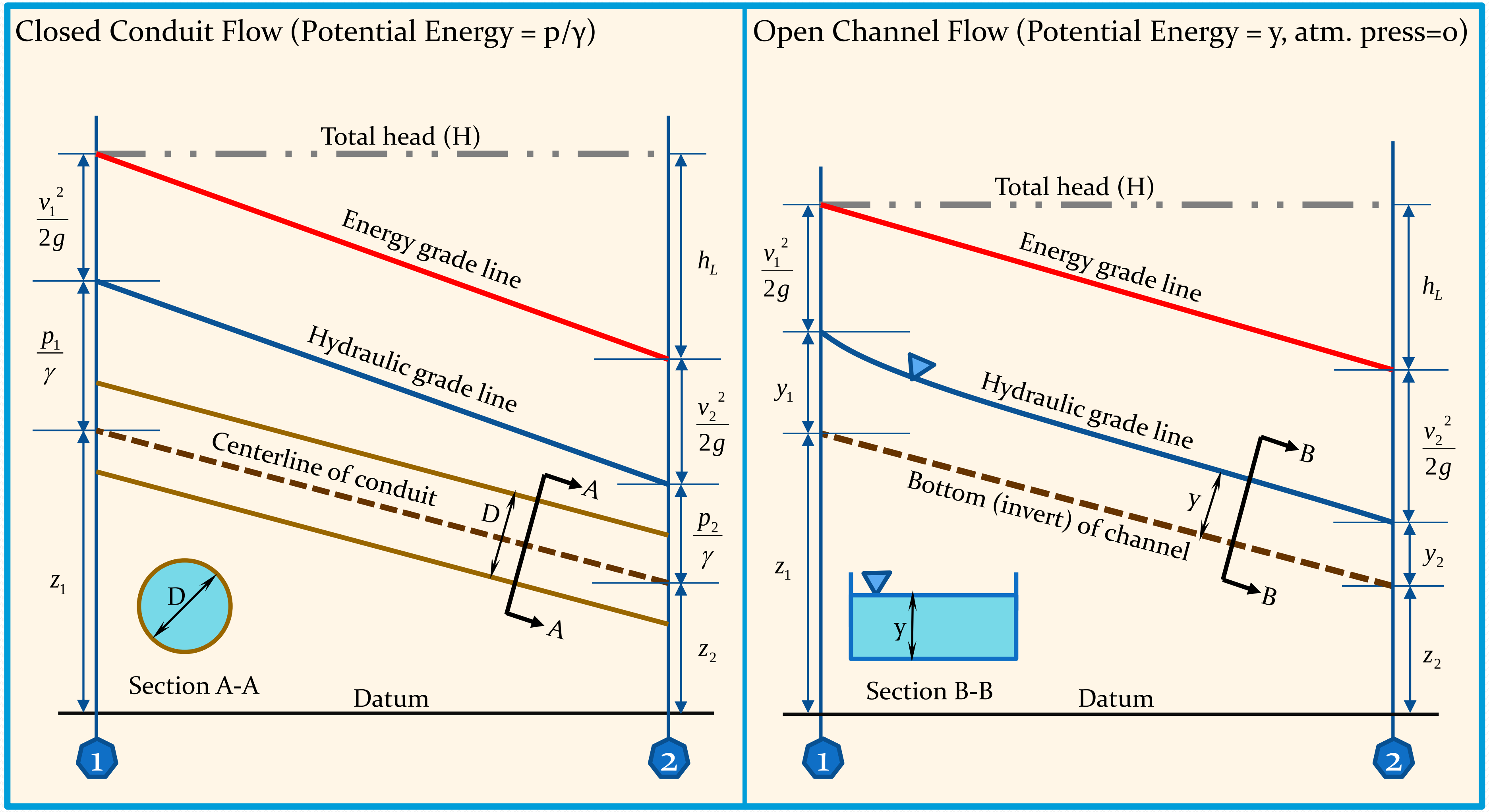
the flow and is lost energy (entropy, Second law of Thermodynamics). In hydraulics, it is commonly referred to as **Friction and Local Losses**, ***h_L***. The Bernoulli (or Energy) Equation between two sections becomes,

$$\boxed{\frac{v_1^{\,2}}{2g} + z_1 + \frac{p_1}{\gamma}[y_1] = \frac{v_2^{\,2}}{2g} + z_2 + \frac{p_2}{\gamma}[y_2] + h_L \quad [1.12]}$$

Terms in brackets [y] describe water depths when dealing with a free-surface (atmospheric pressure) conveyance (i.e., open-channel flow)

Conservation of Energy

Bernoulli Equation



- **Darcy-Weisbach Equation**: It was developed to determine the pressure loss in piping systems in laminar or turbulent flow

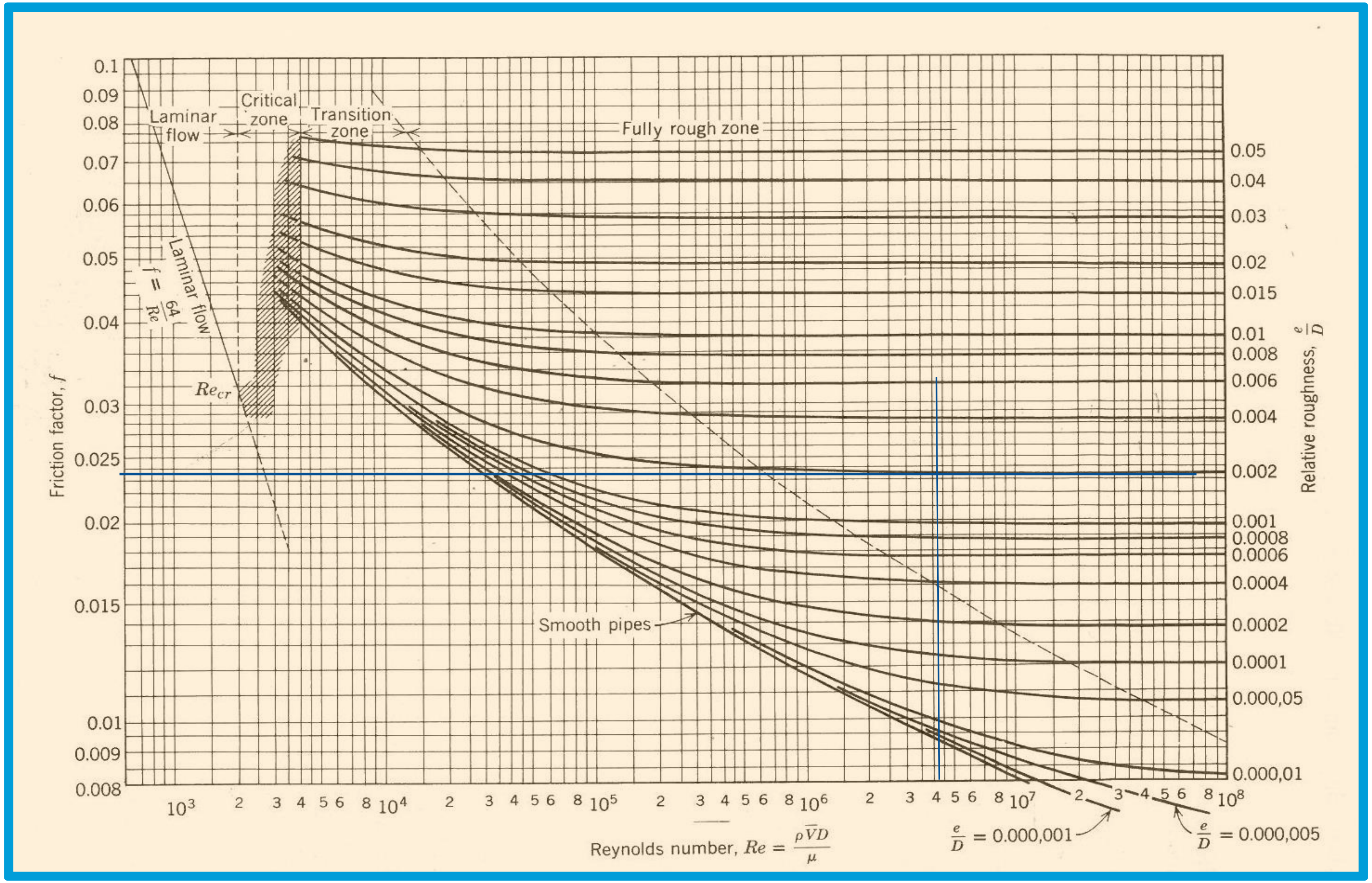
$$\boxed{h_f = f\,\frac{L}{D}\,\frac{v^2}{2g} \qquad [1.13]}$$

h_f = Friction Losses (ft) *f = Friction Factor* *L = Pipe Length (ft)*
v = Flow Velocity (ft/sec) *D = Pipe Diameter (ft)* *g = Gravity Acceleration (32.17 ft/sec²)*

The *f* value is obtained from the Moody Diagram as a function of the *Reynolds Number*, $\boldsymbol{R_e = v\,D/\upsilon}$ ($\boldsymbol{\upsilon}$ is the kinematic viscosity of water in *ft²/sec*). $\boldsymbol{\upsilon = \mu/\rho} \rightarrow \boldsymbol{\mu}$: dynamic viscosity (lb-s/ft^2)

Friction Losses

Moody Diagram



- Any change in the direction of the flow as well as in cross-section area of the flow will create additional losses, and they are known as local losses. Hydraulic features include elbows, curves, fittings, expansions, contractions, drops, valves, gates, etc. They are generally estimated as follows

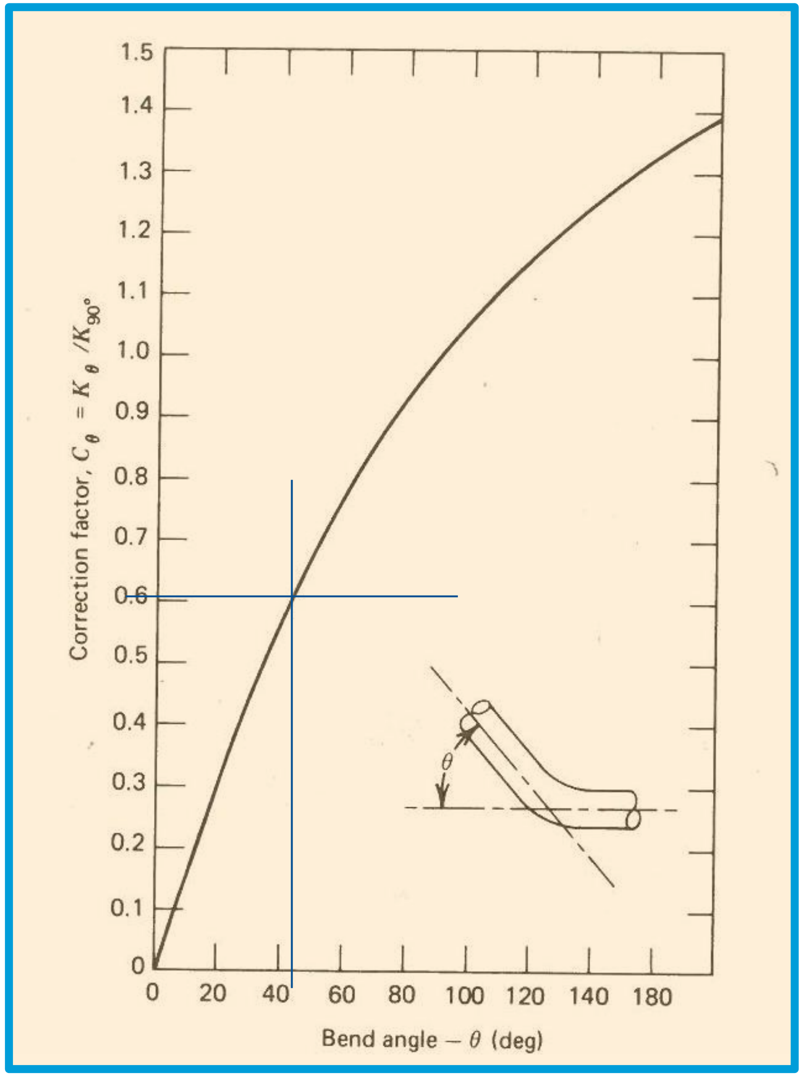
$$\boxed{h_l = K\frac{v^2}{2g} \quad [1.14]} \qquad \boxed{h_l = K\frac{\left|v_1^{\,2} - v_2^{\,2}\right|}{2g} \quad [1.15]}$$

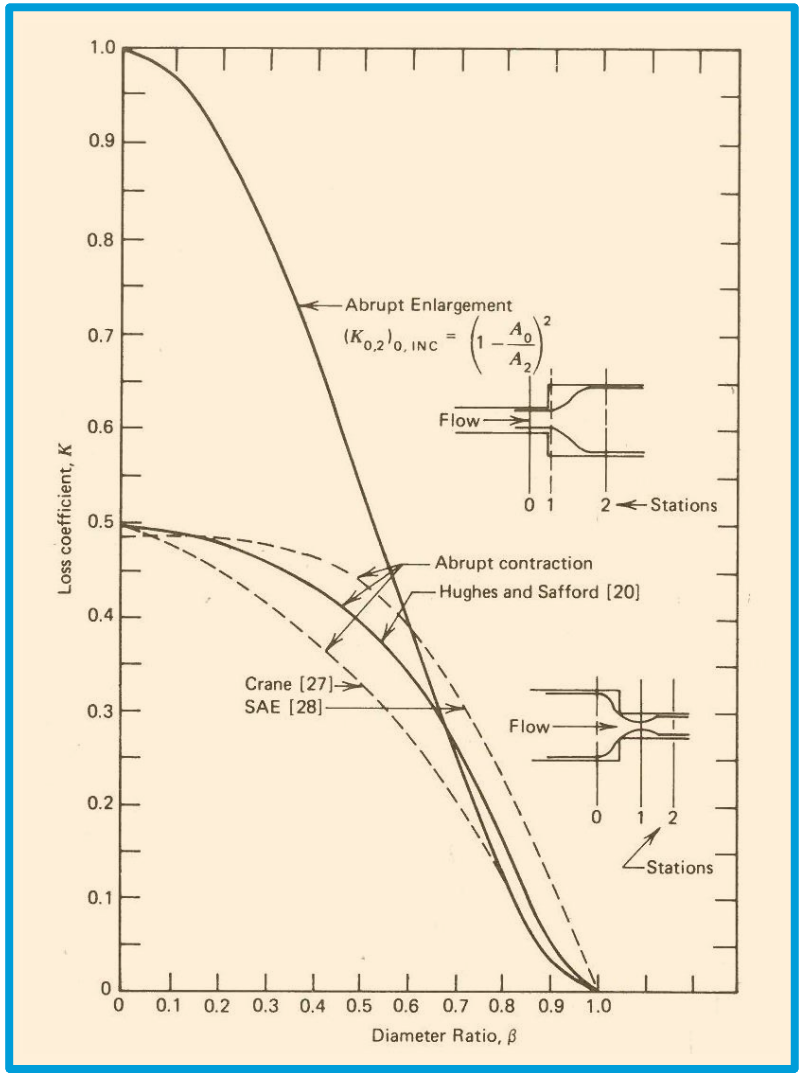
$$K = Dimensionless\ Coefficient$$

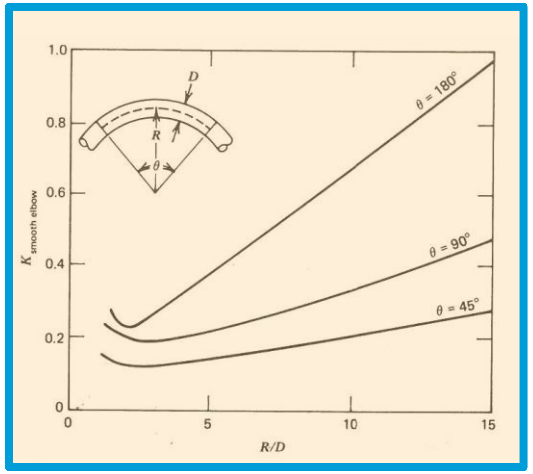
Eq. 1.15 applies when the hydraulic feature has two different flow velocities

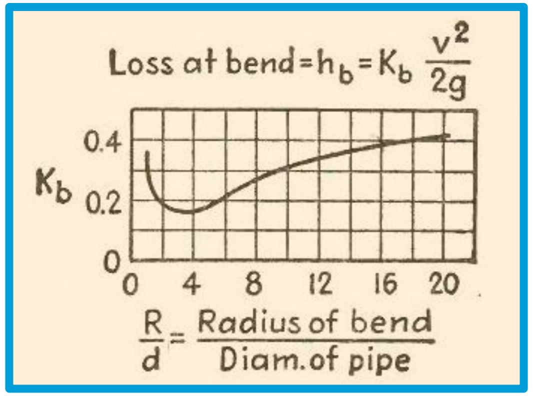
Local Losses

Coefficients of Local Losses









Type of Valve	K_{valve}(open)	
	Screwed	Flanged
Globe	10	5
Angle	5	2
Check	3	—
Butterfly	0.5	—
Gate	0.2	0.1

- **Manning Equation**: It is one of the equations for uniform flow that considers depth, flow area, velocity and discharge for every section of a channel reach constant. Therefore the slope of channel is equal to the slopes of the energy grade line and hydraulic grade line

$$\boxed{Q = \frac{1.49}{n} A R_h^{2/3} \sqrt{S} \quad [1.16]}$$

Q = Flow Rate or Discharge (ft^3/sec)
n = Roughness Coefficient ($sec/ft^{1/3}$)
A = Flow Area (ft^2)

R_h = Hydraulic Radius = A/P_w (ft)
P_w = Wetted Perimeter (ft)
S = Slope of Channel

Friction Losses in Gradually Varied Flow

- **Uniform flow** is always steady, since unsteady uniform flow does not exist in practicality. In natural streams, even steady uniform flow is rare (constant mean velocities) and their stable patterns can be attained only when "boundary layer" is fully developed
- **Manning Equation**, however, is widely applicable to obtain water surface profiles for gradually varied flow either subcritical or supercritical in channels (*Froude Number*, ***F***<1 and ***F***>1, respectively). $\boldsymbol{F} = \boldsymbol{v}/(\boldsymbol{g}\ \boldsymbol{L_F})^{1/2}$ where $\boldsymbol{L_F} = \boldsymbol{A}/\boldsymbol{T}$ (ft) and $\boldsymbol{T}$ is the *Top Width* (width at the free surface)

Friction Losses in Gradually Varied Flow

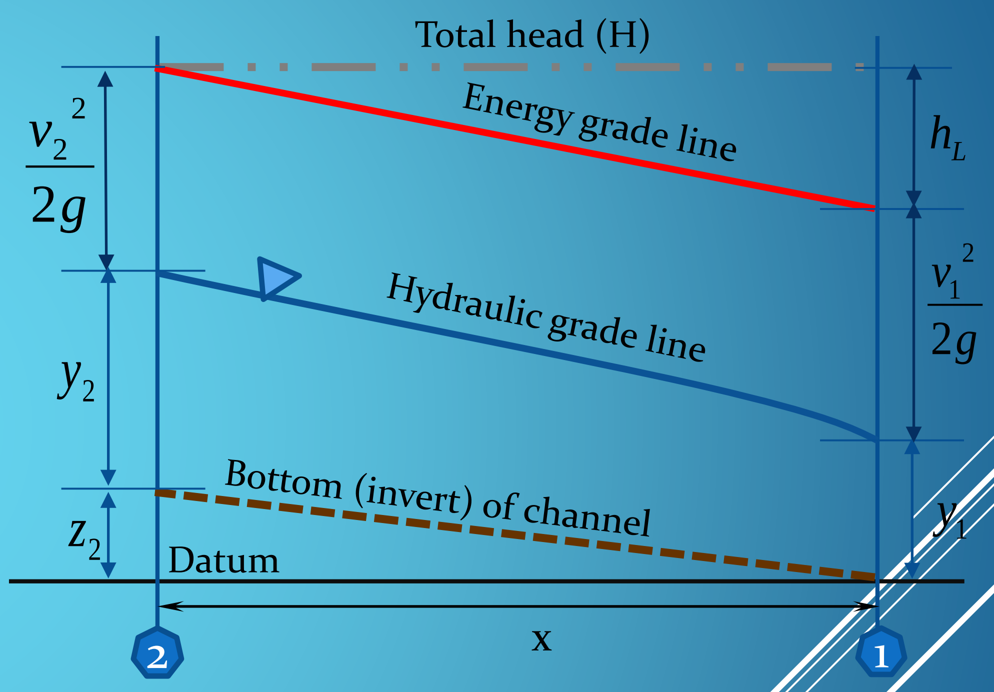
- S from **Manning Equation** becomes the slope of the Energy grade line S_e

$$v = \frac{1.49}{n} R_h^{2/3} S^{1/2}$$

$$\boxed{S_e = \frac{v^2 n^2}{2.22\, R_h^{4/3}} \qquad [1.17]}$$

- Water depth y_1 has to be known (typically subcritical flow is controlled at downstream and supercritical at upstream), so if we are dealing with subcritical flow y_2 is guessed

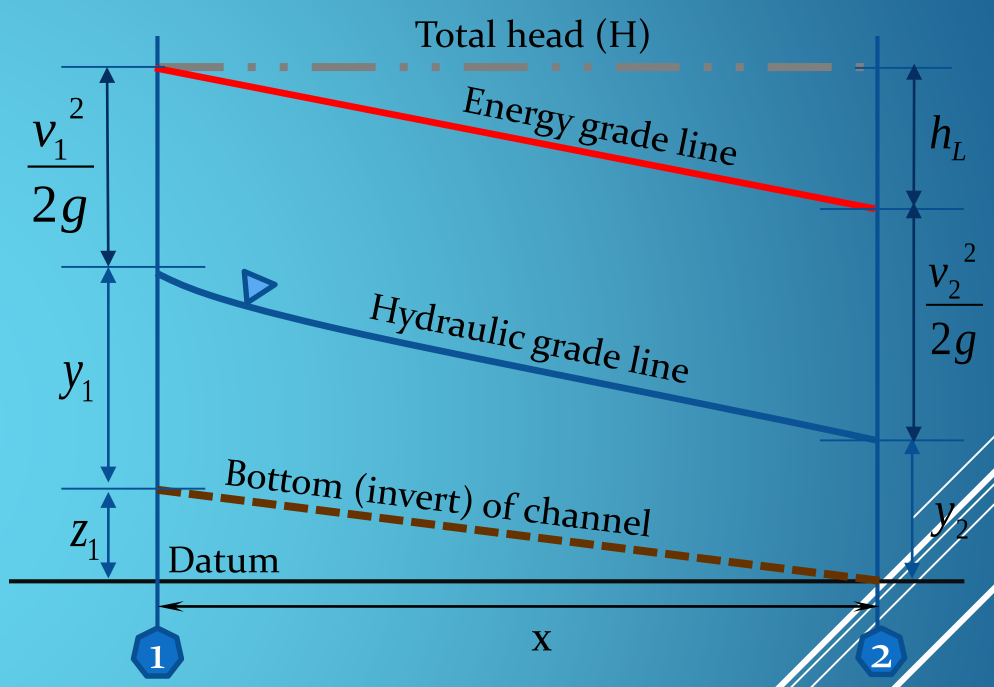
Subcritical flow is controlled downstream



Friction Losses in Gradually Varied Flow

- S_e at sections 1 and 2 are computed from **Manning Equation**, and a trial and error procedure allows to solve for y_2 when Total head (Energy) at 2 equals the known Total head at 1. The average of the two slopes (S_e) multiplied by the distance x between the sections will be h_f (friction losses)
- Sequential sections are taken upstream (subcritical flow) or downstream (supercritical flow) to determine the profile

Supercritical flow is controlled upstream



Friction Losses in Gradually Varied Flow

Florida Department of Environmental Protection

Questions?

Thank you!