Florida Department of Environmental Protection

Hydraulics 3: Problems on Conservation of Momentum and Energy

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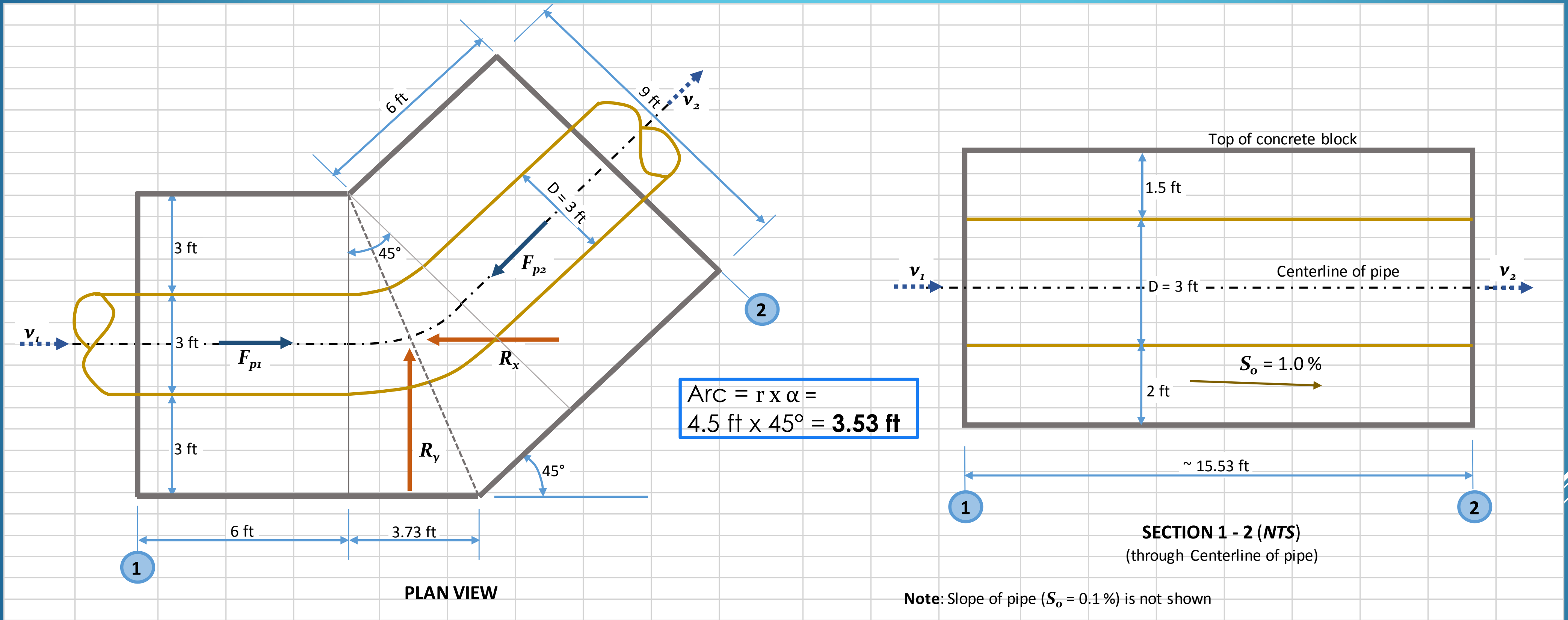
- 10) The shown 45-degree elbow in a 36-in pipe has the depicted dimensions of the concrete block. The following flow conditions are known: Q = 100 ft^3/s, p_1=70.16 psi; K = 0.12; μ=1.918x10^{-5} lb-s/ft^2(dv) ; ρ=1.935 lb-s^2/ft^4 at 75 °F



Conservation of M & E Problem

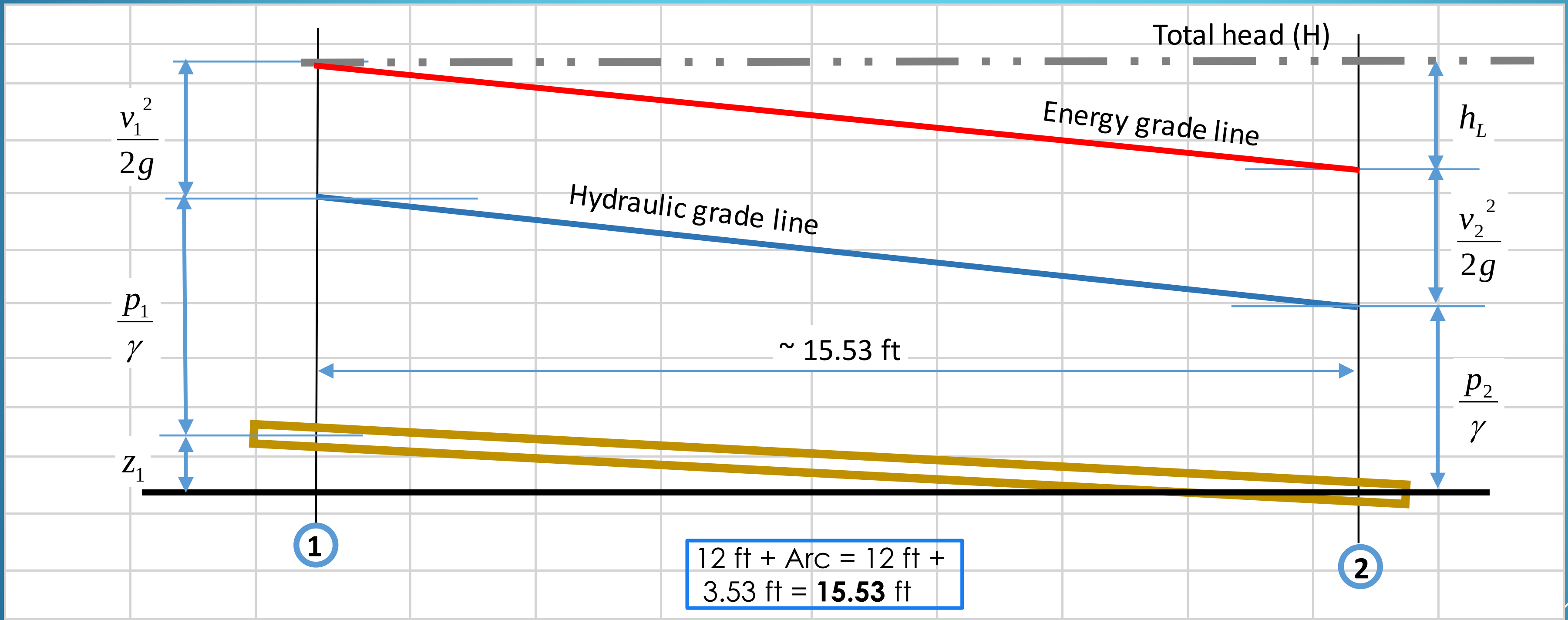
e/D (pipe) = 0.002, γ_w = 62.4 lb/ft^3, $\gamma_{concrete}$ = 150 lb/ft^3

μ_f= 0.5 (friction factor between concrete and soil)

10.1) Calculate the losses though the elbow (both friction and local) and p_2

10.2) Compute the hydrodynamic force in the elbow

10.3) Is the anchor block stable?



Conservation of M & E Problem

10.1) $A_1 = A_2 = \pi\,(3\text{ ft})^2/4 = 7.07\text{ ft}^2$ $v_1 = v_2 = Q/A = (100\text{ ft}^3/\text{s})/7.07\text{ ft}^2 = 14.15\text{ ft/s}$

$z_1 = 15.53\text{ ft} \times 0.001 = 0.02\text{ ft}$

$p_1/\gamma = [70.16\text{ lb/in}^2 \times 144\text{ in}^2/\text{ft}^2]/62.4\text{ lb/ft}^3 = 161.91\text{ ft}$

$v_1^2/2g = v_2^2/2g = (14.15\text{ ft/s})^2 / (2 \times 32.17\text{ ft/s}^2) = 3.11\text{ ft}$

From Eq. 1.14,

$$h_l = K\frac{v^2}{2g} = 0.12 \times 3.11\,ft = 0.37\,ft$$

$R_e = v\,D/\upsilon \rightarrow \upsilon$ (kv)$= \mu/\rho = 1.918 \times 10^{-5}\text{ lb-s/ft}^2/1.935\text{ lb-s}^2/\text{ft}^4 = 9.912 \times 10^{-6}\text{ ft}^2/\text{s}$

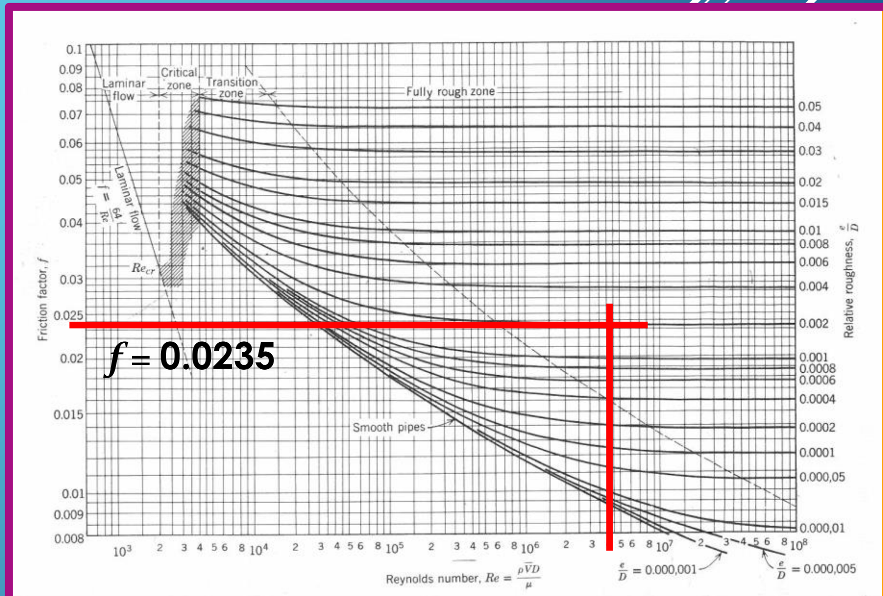
$R_e = 14.15\text{ ft/s} \times 3\text{ ft} / 9.912 \times 10^{-6}\text{ ft}^2/\text{s} = 4.28 \times 10^6$

from Moody Diagram for: $R_e = 4.28 \times 10^6$ and $e/D = 0.002 \rightarrow f = 0.0235$

From Eq. 1.13,

$$h_f = f\frac{L}{D}\frac{v^2}{2g} = 0.0235 \times \left(\frac{15.53\,ft}{3\,ft}\right) \times 3.11\,ft = 0.38\,ft$$

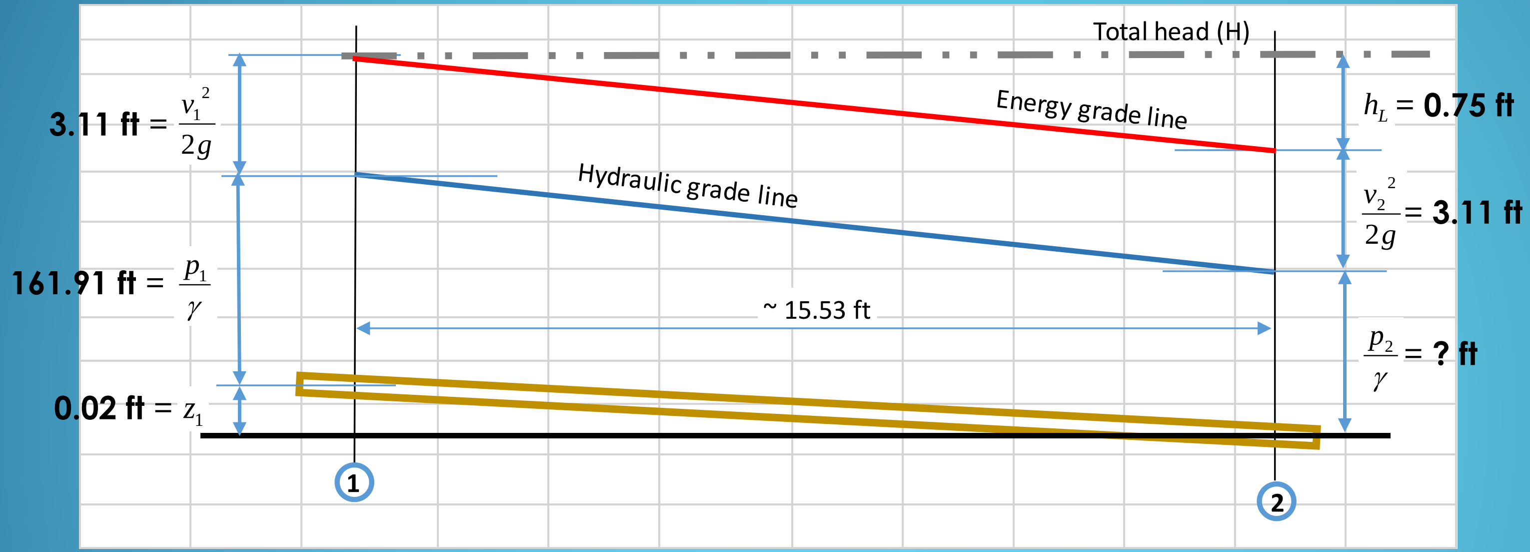
$h_L = h_l + h_f = 0.37\text{ ft} + 0.38\text{ ft} = \mathbf{0.75\text{ ft}}$



Conservation of M & E Problem

From Eq. 1.12,

$$\frac{v_1^{\,2}}{2g} + z_1 + \frac{p_1}{\gamma} = \frac{v_2^{\,2}}{2g} + z_2 + \frac{p_2}{\gamma} + h_L \quad [1.12]$$



3.11 ft + 0.02 ft + 161.91 ft = 3.11 ft + $\boldsymbol{p_2/\gamma}$ + 0.75 ft

$\boldsymbol{p_2/\gamma}$ = 161.17 ft → $\boldsymbol{p_2}$ = 161.17 ft x 62.4 lb/ft^3 / 144 in^2/ft^2 =

$\boldsymbol{p_2}$ = **69.84 psi**

Conservation of M & E Problem

10.2) From Eq. 1.8,

$$\boxed{\Sigma F_x = \rho Q\left(v_{2x} - v_{1x}\right) \quad [1.8]}$$

$F_{p1} - R_x - F_{px}$ = 1.935 lb-s²/ft⁴ x 100 ft³/s (14.15 cos 45° –14.15) ft/s

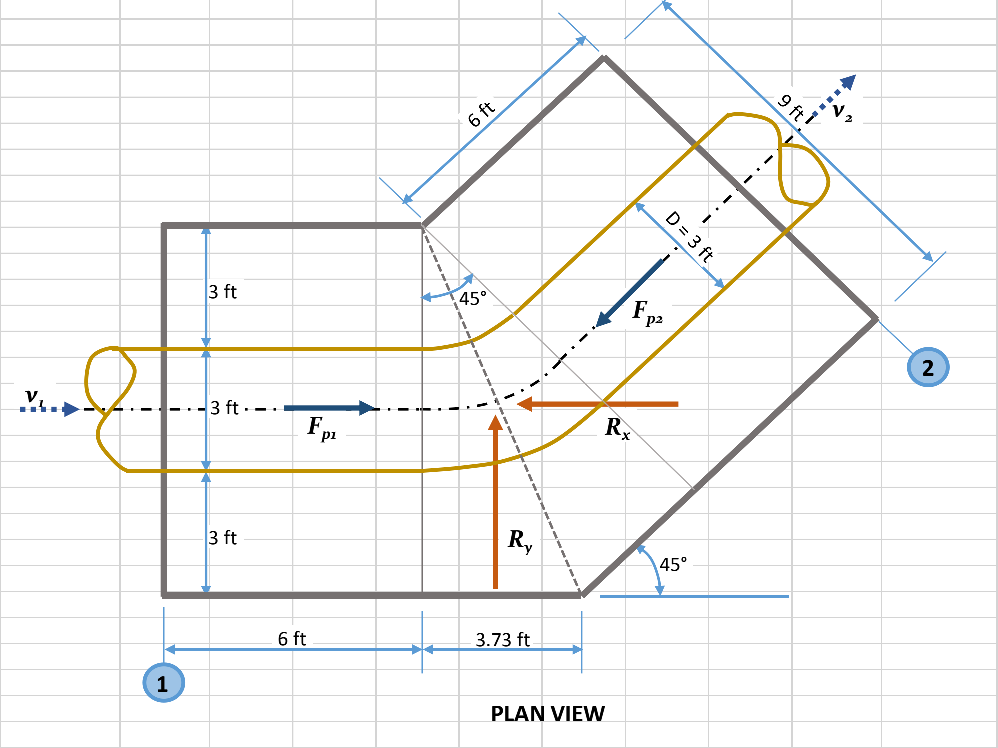
F_{p1} = 70.16 lb/in² x 144 in²/ft² x 7.07 ft² = 71,414 lb

F_{p2} = 69.84 lb/in² x 144 in²/ft² x 7.07 ft² = 71,089 lb

F_{px} = 71,089 lb x cos 45° = 50,268 lb

71,414 lb – R_x – 50,268 lb = – 802 lb

R_x = 71,414 lb – 50,268 lb + 802 lb = **21,948 lb** (–)



Conservation of M & E Problem

10.2) From Eq. 1.9,

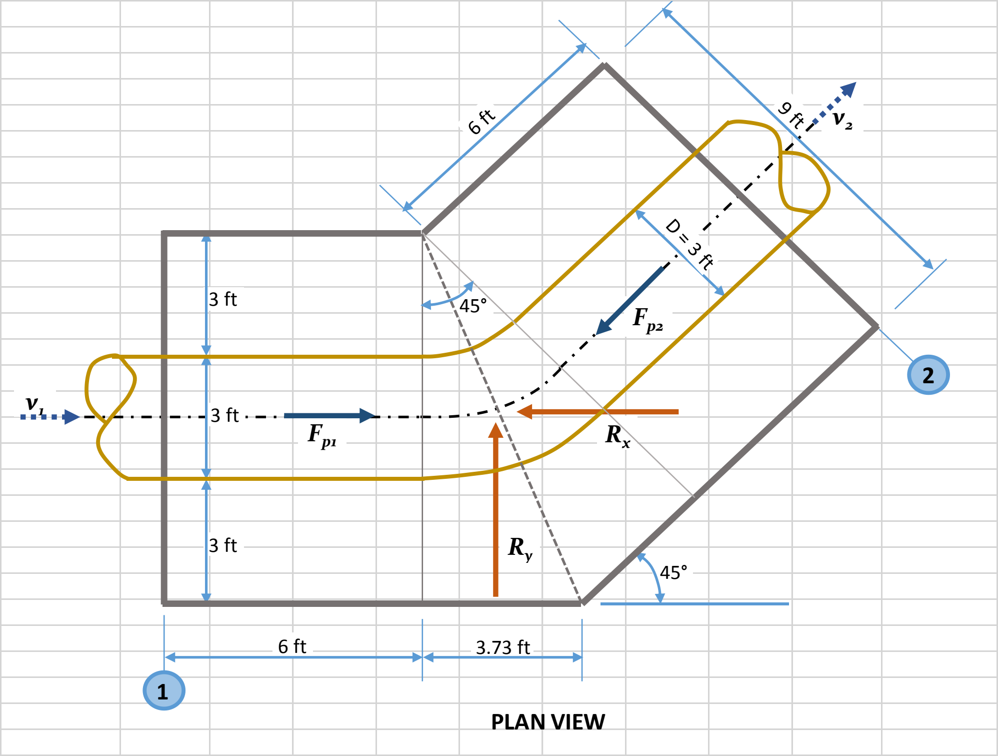
$$\boxed{\Sigma F_y = \rho Q\left(v_{2y} - v_{1y}\right) \quad [1.9]}$$

$R_y - F_{py}$ = 1.935 lb-s²/ft⁴ x 100 ft³/s (14.15 sin 45° – 0) ft/s

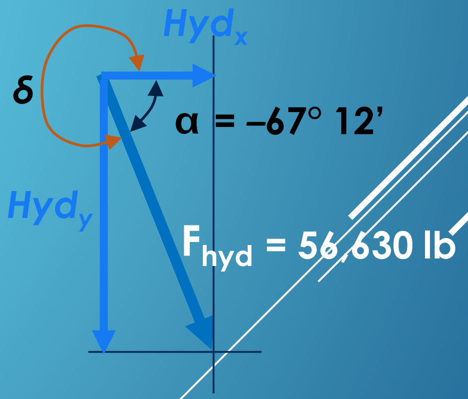
F_{py} = 71,089 x sin 45° = 50,268 lb

R_y – 50,268 lb = 1936 lb → R_y = 50,268 lb + 1936 lb =

R_y = **52,204 lb** (+)



$$F_{hyd} = \sqrt{Hyd_x^{\,2} + Hyd_y^{\,2}} = \sqrt{(21{,}948lb)^2 + (-52{,}204lb)^2} = 56{,}630lb \ \ magnitude$$



α = arc tan (Hyd_y/Hyd_x) = arc tan (–2.37853) = –67.20° = **–67° 12’**

δ = 360° – α = 360° – 67° 12’ = **292° 48’** (292.80°) *direction*

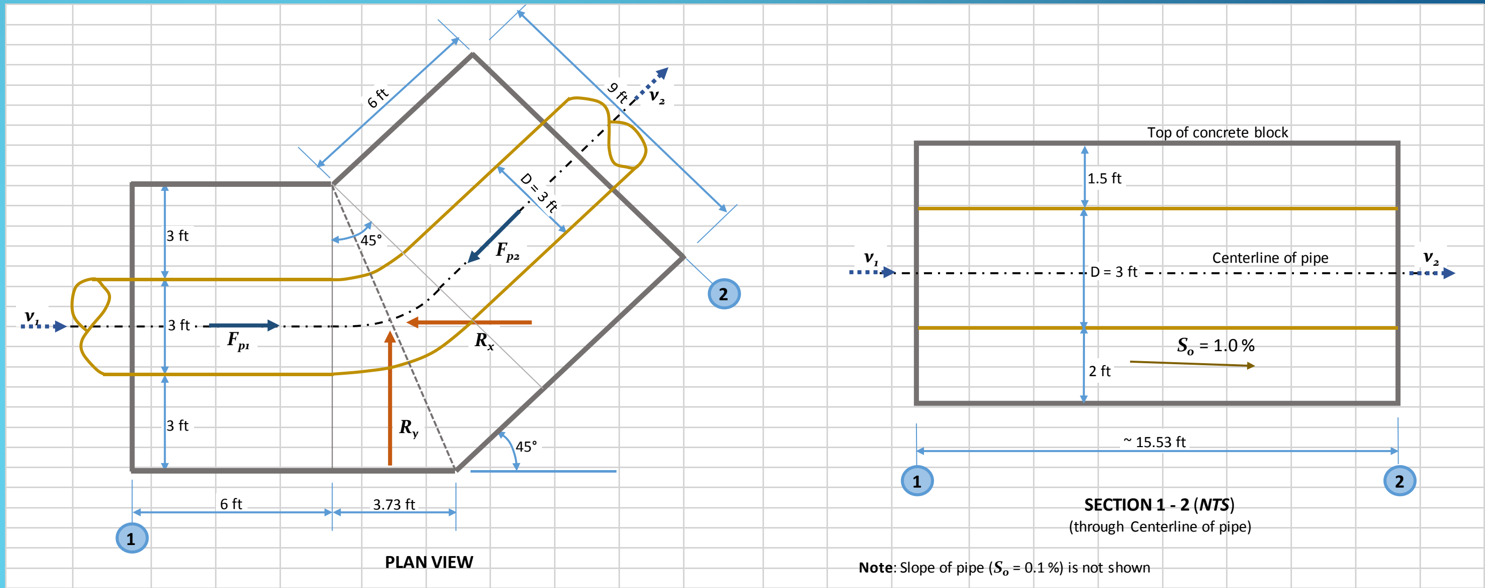
Conservation of M & E Problem

- 10.3) To solve the stability of the concrete block, it is necessary to compute the volumes occupied by concrete and water to estimate the total weight of the block

Surface area of Block =
2 x [(9.73 ft + 6 ft)/2 x 9 ft] = 141.57 ft^2

Volume of Block = 141.57 ft^2 x 6.5 ft =

Volume of Block = 920.21 ft^3



Volume of Pipe (water) =
π (3 ft)^2/4 x (15.53 ft) = **109.78 ft^3**

Volume of Concrete = 920.21 ft – 109.78 ft =
Volume of Concrete = **810.43 ft^3**

Weight of Block (water + concrete) = 109.78 ft^3 x 62.4 lb/ft^3 + 810.43 ft^3 x 150 lb/ft^3

Weight of Block (water + concrete) = 6850 lb + 121,564 lb = **128,414 lb**

Resisting Force = F_{frict} = μ_f x W = 0.50 x 128,414 lb = **64,207 lb**

The force producing sliding is the hydrodynamic force, F_{hyd} = **56,630 lb** (acting) which is smaller than F_{frict} = **64,207 lb** (resisting), therefore the block is stable to sliding

Conservation of M & E Problem

- 11) A channel has a transition 200 ft long from trapezoidal cross section to rectangular cross section. The total length is 2400 ft. The dimensions of the trapezoidal and rectangular canal (upstream and downstream, respectively) are given in the following sketch

Q = 895 ft^3/s S = 0.0005 n = 0.013

Invert Elev at C/S 1 = **17.94 ft NAVD**

y_1 = **7.35 ft** (control, Subcritical flow)

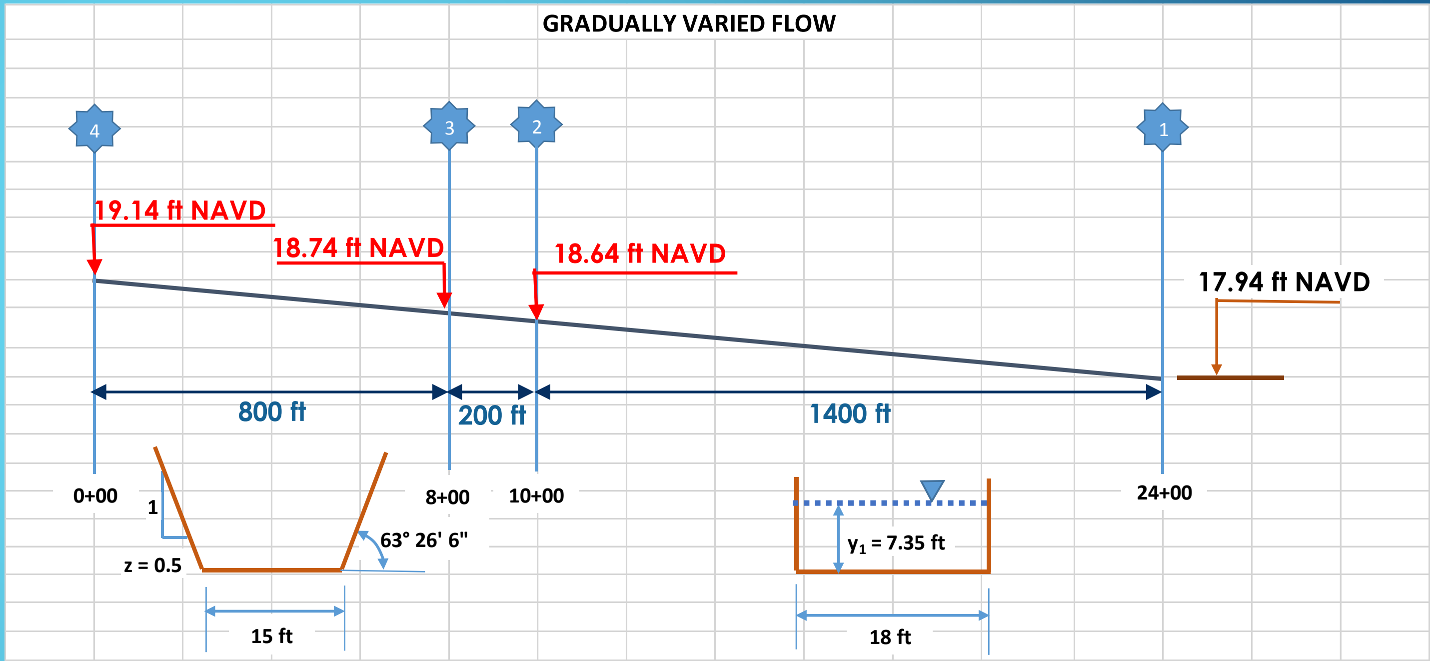
Trapezoidal Canal

B = 15 ft z = 0.50

Rectangular Canal

B = 18 ft

Flow conditions are known at C/S 1 (it is referred as to boundary condition to solve the ***Bernoulli Equation*** for gradually varied flow conditions.



The following table illustrates the best way for solving these problems. **Compute the water depths (or water profile) at C/S 2, 3 and 4**

Conservation of Energy Problem

Conservation of Energy Problem

Solution to Problem 11. Subcritical flow with $y_1 = 7.35$ ft (control) ***GVF***

C/S	y (ft)	L (ft)	T (ft)	A (ft²)	v (ft/s)	$v^2/2g$ (ft)	P_w (ft)	R_h (ft)	L_F (ft)	F	S_e	avg S_e	K	h_l (ft)	h_f (ft)	h_L (ft)	z (ft)	$H-h_l$ (ft)
1	7.35	0	18.00	132.30	6.76	0.71	32.70	4.05	7.35	0.44	0.00054	–	–	–	–	–	17.94	26.00
2	7.41	1400	18.00	133.38	6.71	0.70	32.82	4.06	7.41	0.43	0.00053	0.00053	–	–	0.75	0.75	18.64	26.00
3	6.56	200	21.56	119.92	7.46	0.87	29.67	4.04	5.56	0.56	0.00066	0.00059	0.30	0.05	0.12	0.17	18.74	26.00
4	6.10	800	21.10	110.11	8.13	1.03	28.64	3.84	5.22	0.63	1.1E-05	0.00033	–	–	0.27	0.27	19.14	26.00

B C D A E

$T = B + 2 \times y \times z$

$A = (B + zy) \times y \quad [2.8]$

$P_w = B + 2y\sqrt{1+z^2} \quad [2.9]$

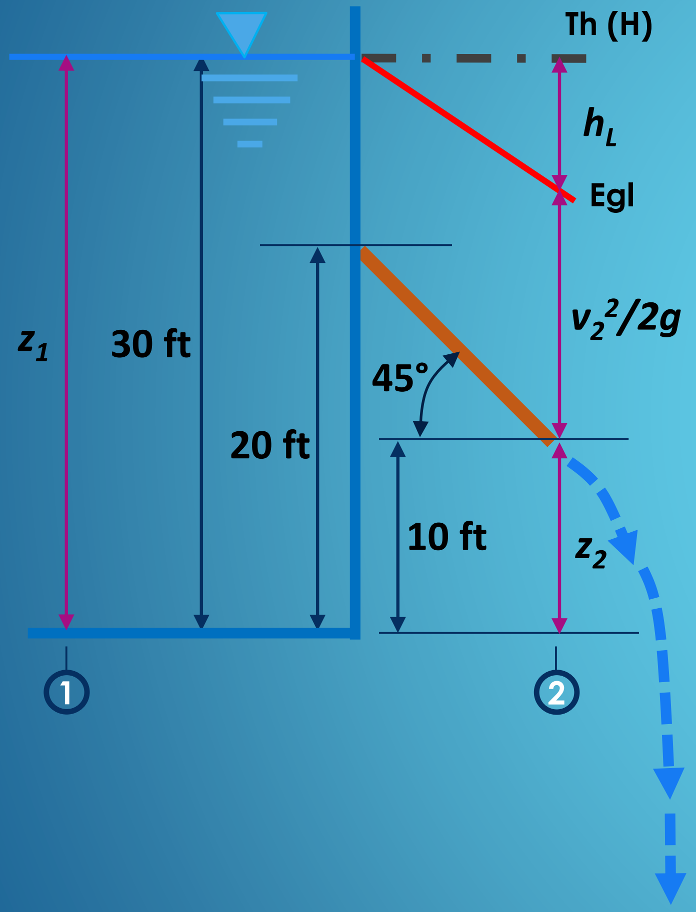
$F = \dfrac{v}{\sqrt{g \times L_F}}$

$S_e = \dfrac{v^2 n^2}{2.22 R_h^{4/3}} \quad [1.17]$

The application of ***Bernoulli Equation*** for subcritical flow (controlled downstream) computes the water profile advancing upstream,

E = A + B + C – D

- 12) Estimate the flow discharge through a 9-in pipe connected to a tank with the dimensions shown in the sketch. The pipe is smooth with $f = 0.011$



Length of pipe = 10 ft/sin 45° = 14.14 ft

$z_1 = z_2 + v_2^2/2g + h_L$

$A = \pi$ (0.75 ft)^2 /4 = 0.44 ft^2

$v_2^2/2g = (Q/A)^2/2g = Q^2/(A^2\, 2g) =$

$Q^2/[(0.44\text{ ft})^2 \times 2 \times 32.17\text{ ft/s}^2] =$
$Q^2/12.56 = 0.07963\, Q^2$

from Darcy-Weisbach Equation [1.13],

$$h_f = f\,\frac{L}{D}\,\frac{v^2}{2g} = 0.011\,\frac{14.14\,ft}{0.75\,ft}\,\frac{Q^2}{12.57} = 0.01652\,Q^2$$

30 ft = 10 ft + 0.07963 Q^2 + 0.01652 Q^2

Q^2 = 20 ft / 0.09615 s^2/ft^5 = 208.01 ft^6/s^2

Q = **14.42 ft^3/s**

Conservation of Energy Problem

- The conversion of hydraulic energy (both kinetic and potential) to electric energy represents a valuable renewable resource. Hydropower developments are of great scope and involve the design, operation and maintenance of many and diverse structures, machines, processes and systems
- With a well implemented infrastructure, it achieves efficient and economical production and is significantly safe to the environment
- The hydropower generation equation derives from the conservation of energy and is calculated as follows,

$$\boxed{P = \gamma \times Q \times H \quad [3.1]}$$

Eq. 3.1 can be expressed in horsepower (***HP***) and kilowatt (***KW***) units as follows,

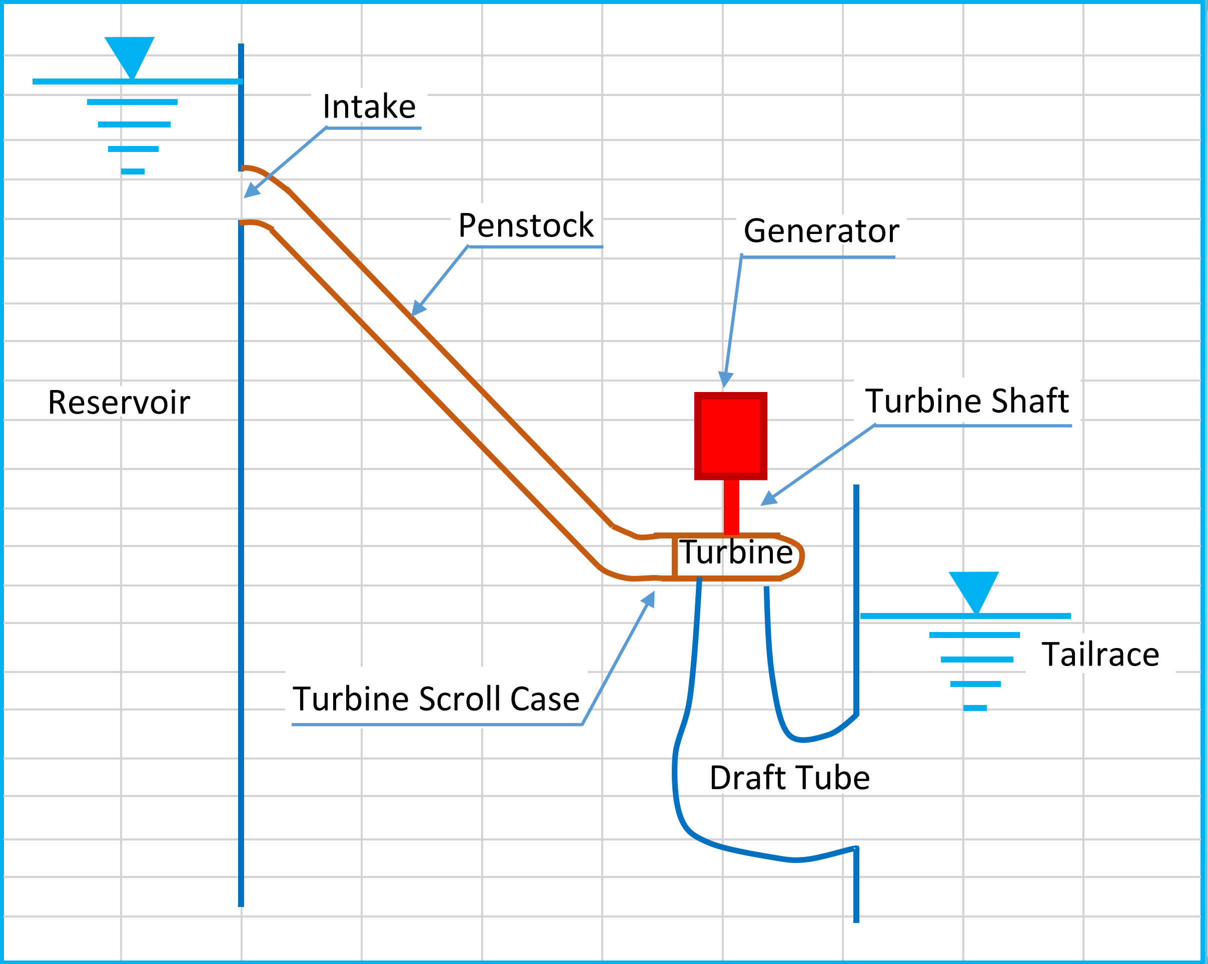
$$\boxed{P_{HP} = \frac{\gamma \times Q \times H}{550} = \frac{Q \times H}{8.81} \quad [3.2]} \qquad \boxed{P_{KW} = \frac{0.746 \times \gamma \times Q \times H}{550} = \frac{Q \times H}{11.814} \quad [3.3]}$$

$\boldsymbol{P_{HP}}$ and $\boldsymbol{P_{KW}}$ represent the power being transmitted by a flow discharge $\boldsymbol{Q}$ (ft^3/s) under a total head available $\boldsymbol{H}$ (ft). The following sketch illustrates the components of a hydropower plant

Hydropower Generation

Hydropower Generation

Typical Components of a Hydropower Plant



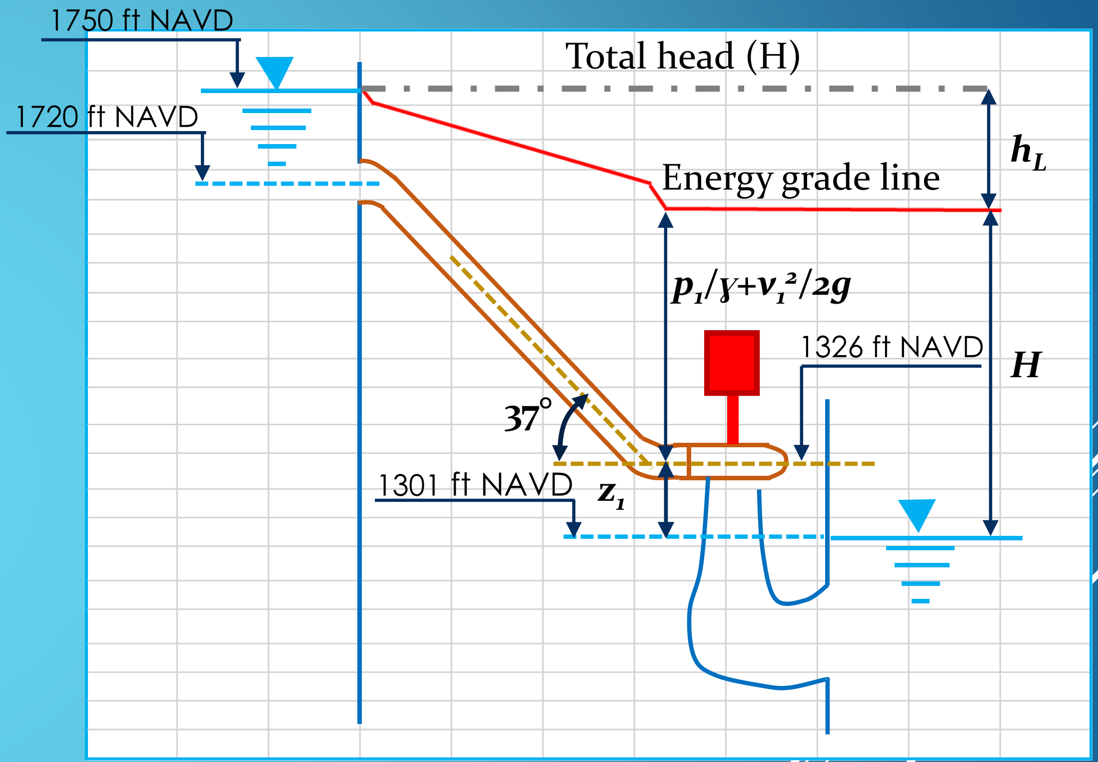
Largest Hydropower Plant

Dam	Country	River	Installed Capacity (MW) [(1)]	Annual electricity production (TW-hr) [(2)]	Area flooded (Km^2)	Years of completion
Three Gorges	China	Yangtze	22,500	98.1	632	2003/2012
Itaipu	Brazil Paraguay	Paraná	14,000	98.6	1,350	1984/1991, 2003
Guri	Venezuela	Caroní	10,235	53.4	4,250	1978, 1986

[(1)] MW (megawatt) = 10^6 Watts
[(2)] TW (terawatt) = 10^{12} Watts

- 13) Compute the power produced by a hydropower plant (***HP*** and ***KW***) with the following characteristics:

HW = 1750 ft NAVD
TW = 1301 ft NAVD
Elev. Intake = 1720 ft NAVD
Eff turbine = 0.83
K = 0.7 (intake + entrance)
Q = 788 ft^3/s
D = 5.0 ft
Elev. Turbine = 1326 ft NAVD
Eff generator = 0.91
K = 0.35 (elbow)
ν = 9.912E-06 ft^2/s @ 75° F → R_e = 2.024E+07
For smooth pipe, from Moody diagram → f = 0.008



Length of Penstock = (1720 – 1326) ft / sin 37° = 654.69 ft

A = π (5.0 ft)^2/4 = 19.63 ft^2

v = (788 ft^3/s) / 19.63 ft^2 = 40.13 ft/s

$v^2/2g$ = (40.13 ft/s)^2 / (2 x 32.17 ft/s^2) = 25.03 ft

h_l = 1.05 x 25.03 ft = 26.28 ft

h_f = 0.008 x (654.59 ft/5 ft) x 25.03 ft = 26.22 ft

$h_L = h_l + h_f$ = 26.28 ft + 26.22 ft = 52.50 ft

Hydropower Generation

$H = HW - TW - h_L$ = 1750 ft – 1301 ft – 52.50 ft = 449 ft – 52.50 ft = 396.50 ft

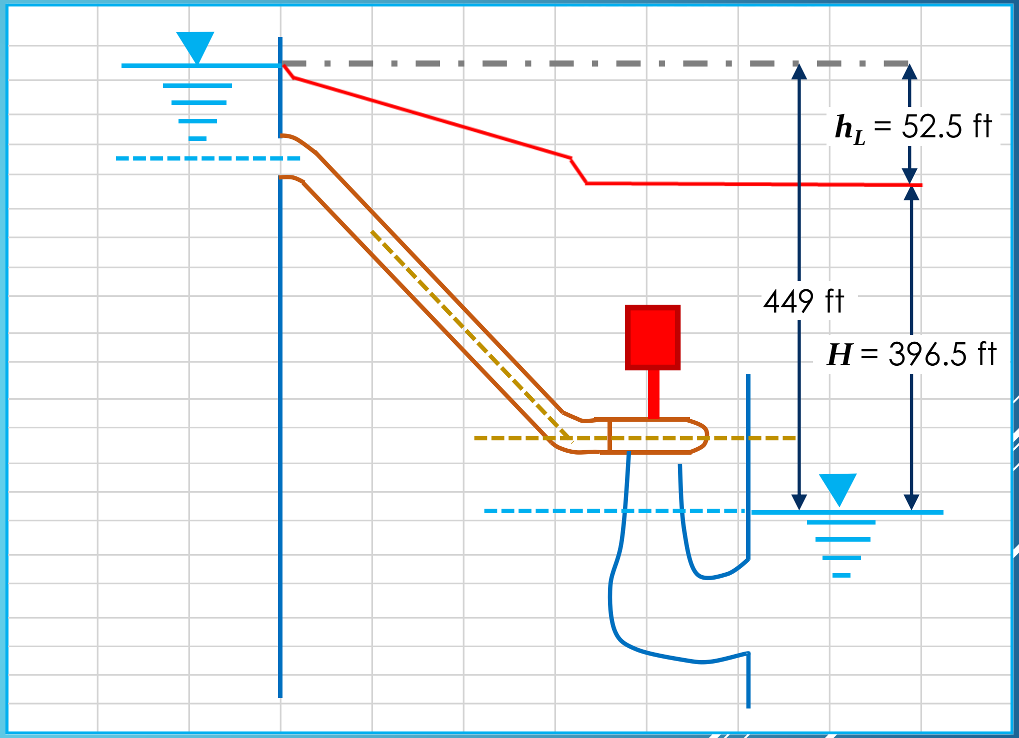
from Eq. 3.2,

$$P_{HP} = \frac{Q \times H}{8.81} = \frac{788\ ft^3/s \times 396.5\ ft}{8.81} = 35{,}464\,HP$$

from Eq. 3.3,

$$P_{KW} = \frac{Q \times H}{11.814} = \frac{788\ ft^3/s \times 396.5\ ft}{11.814} = 26{,}446\,KW$$

The electrical power produced by the plant is at the generator (and even sometimes affected by the transformer as well)



Eff_{plant} = 0.83 x 0.91 = 0.76

P_{HP} = 35,464 HP x 0.76 = **26,768 HP**

P_{KW} = 26,446 HP x 0.76 = **19,975 KW**

Hoover Dam has currently an installed capacity of 2080 MW (17-unit plant). Turbines range from 130 MW (13) to 61.5 (1)

Hydropower Generation

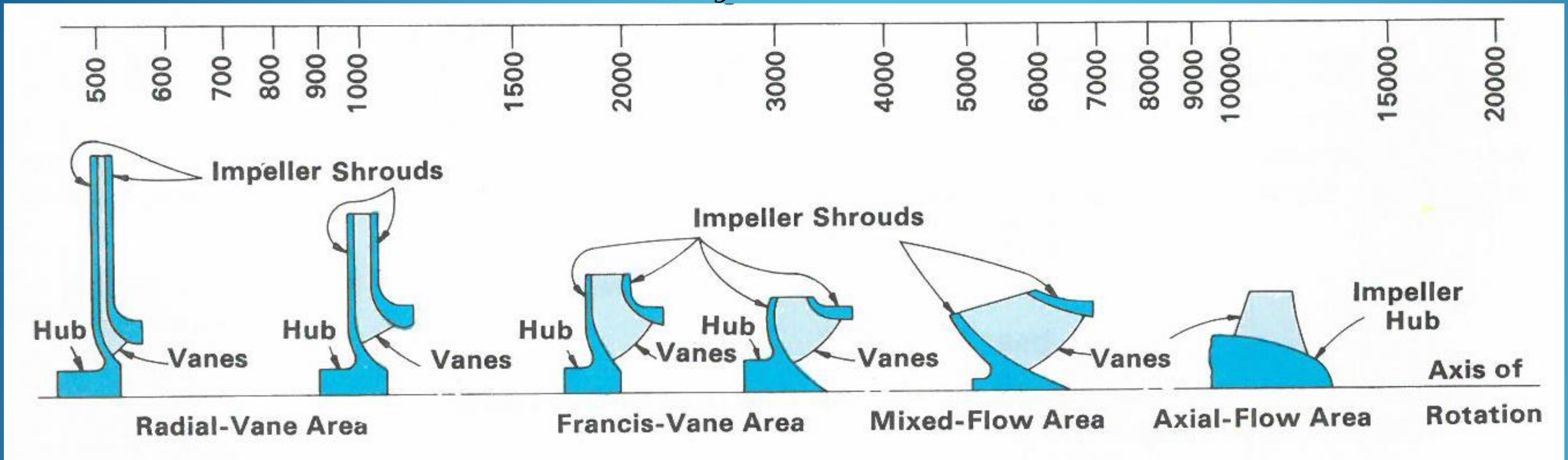
- Pumps are an integral part of pumping systems and they are also often used in drainage, irrigation and other areas associated with water resources
- A main part of their design is based on the Energy Equation as with turbines. Pumps perform in a reverse mode in relation to turbines. They create hydraulic energy when this is not available by means of mechanical energy (i.e., electrical or diesel engines)
- There are three main classes of pumps: Centrifugal, Rotary, and Reciprocating
- Centrifugal pumps are more widely used and typed by their mechanical configuration that depends on the impeller or casing
- The selection of an appropriate centrifugal pump is obtained by the ***Specific Speed*** (N_S) which determines the three types of impeller shapes associated with radial, mixed and axial flows. N_S values are calculated as follow,

$$\boxed{N_S = \frac{N \times \sqrt{Q}}{H_T^{3/4}} \quad [3.4]}$$

where N = speed (rpm), Q = flow discharge at optimum efficiency (gpm), and H = total head (ft)

Pumps

N_S values



- 14) An impeller will operate at 580 rpm for an optimum flow discharge of 50 ft^3/s against a total head of 7.5 ft. Determine the type of centrifugal pump for this application

448.831 gpm/cfs

Q = 50 ft^3/s x (7.48052 gal/ft^3) x (60 s/min) = 22,441.6 gpm

From Eq. 3.4,

$$N_S = \frac{N \times \sqrt{Q}}{H_T^{3/4}} = \frac{580 \times \sqrt{22,441.6}}{7.5^{3/4}} = 19,172$$ Axial-Flow type

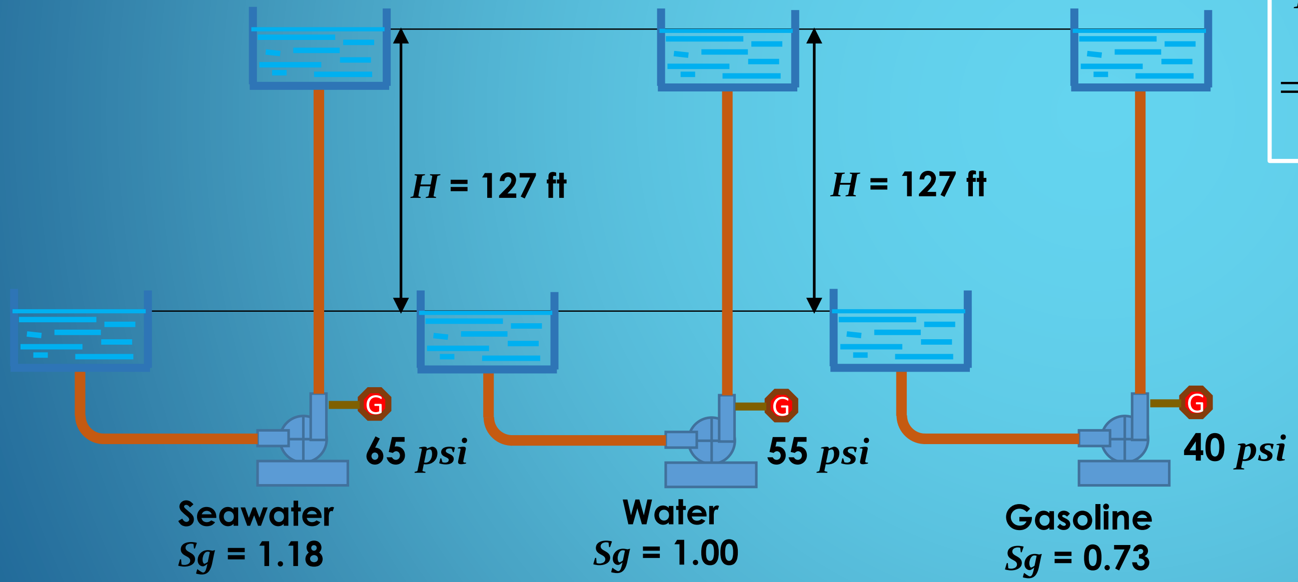
Pumps

- The head (H) developed by a centrifugal pump depends on the peripheral velocity (u) of its impeller and it is expressed by,

$$H = \frac{u^2}{2g} \quad [3.5]$$

- It is important to realize that the head (H) developed by a pump does not vary with the weight of the liquid pumped

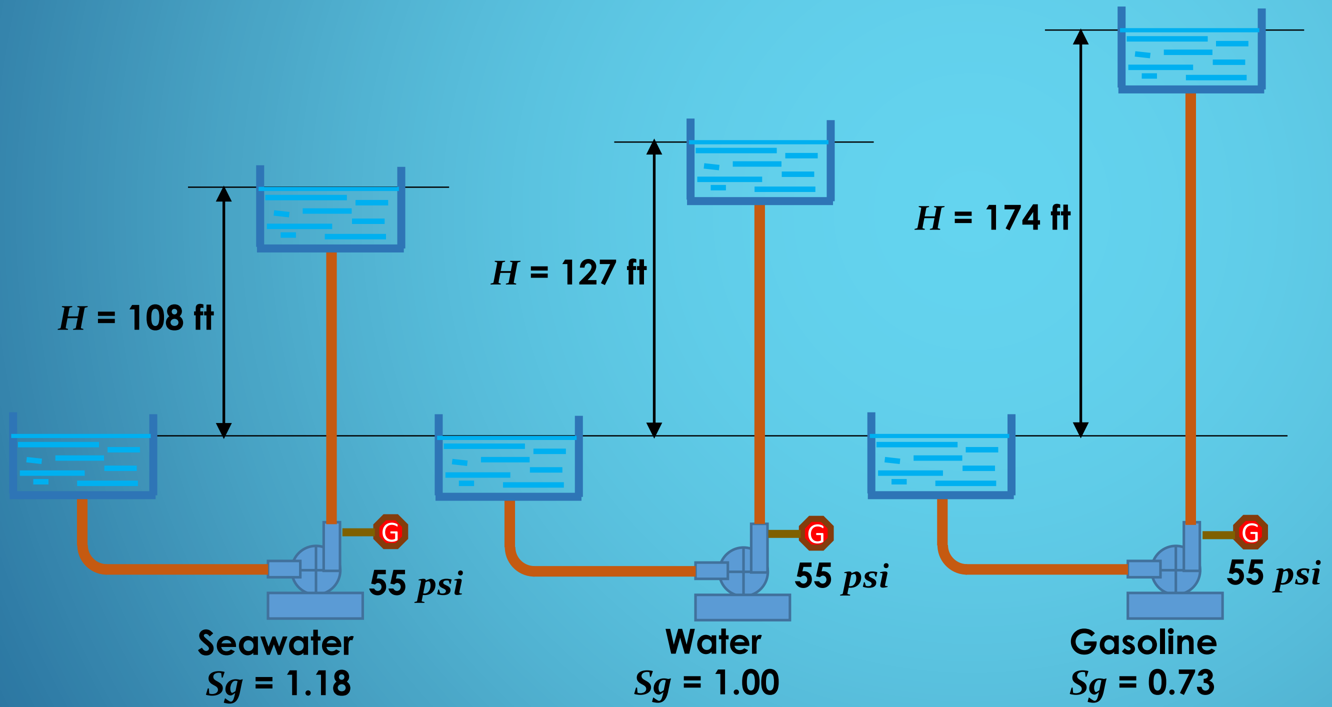
$$p(psi) = H(ft) \times Sg \times \gamma(lb/ft^3)/144\,in^2/ft^2 = \\ = \frac{H(ft) \times Sg}{144\ ft-in^2/62.4\,lb} = \frac{H \times Sg}{2.308}(psi) \qquad [3.6]$$



Pumps: Specific Gravity and Head

- If the three pumps operate at 55 psi and the weight of the three liquids is different, the liquids are pumped at different elevations. The lighter the liquid is (i.e., smaller ***Sg***), the higher the head (***H***) at which the liquid is pumped
- From Eq. 3.6,

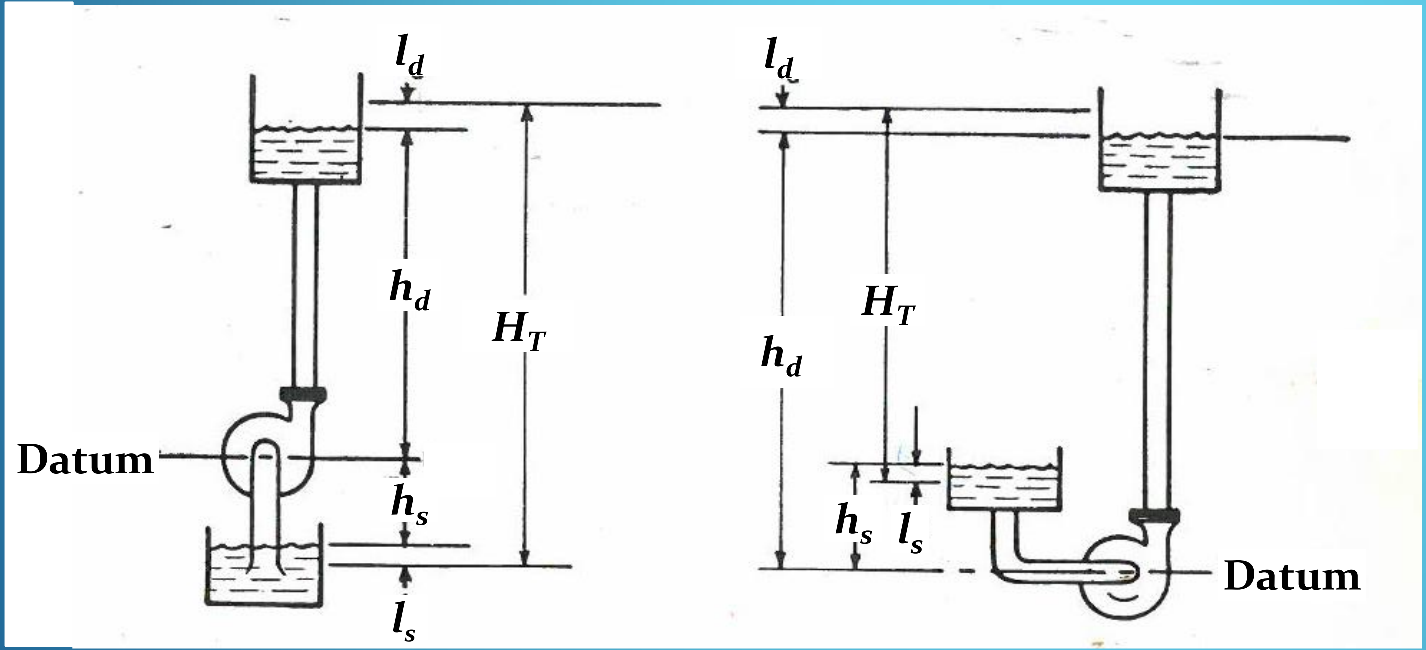
$$p\,(psi) = \frac{H \times Sg}{2.308}\,(psi) \Rightarrow H = \frac{p\,(psi) \times 2.308}{Sg} \qquad (3.7)$$



Pumps: Specific Gravity and Head

Pumps: Suction Lift and Suction Head

- Pumps operate, as shown in the below sketch, with suction lift or suction head
- When operation with suction lift, the level of the water supply is below the centerline of the pump. A pressure gauge at the suction section (flange) would display a negative pressure (or vacuum)
- If the level of the water supply is above the centerline of the pump, the pump operates with suction head



H_T = Total Head (ft)

h_d = Static Discharge Head (ft)

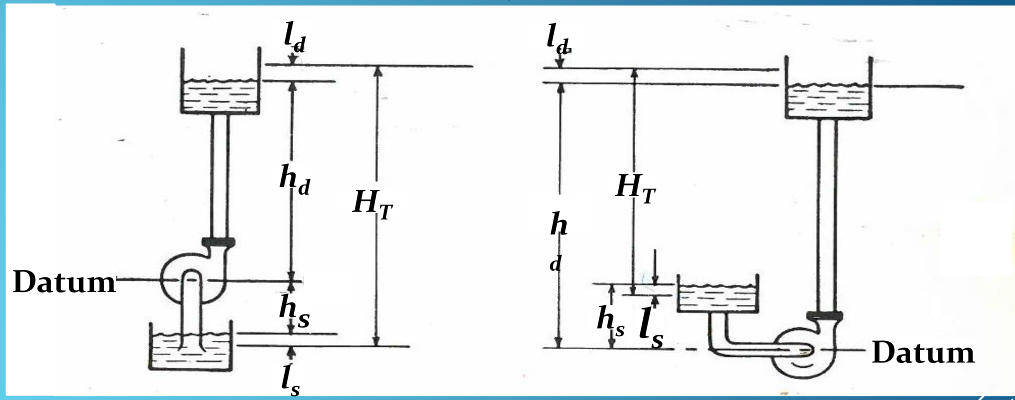
h_s = Static Suction Lift or Head (ft)

l_d = Total Losses in Discharge, local + friction (ft)

l_s = Total Losses in Suction, local + friction (ft)

Pumps: Suction Lift and Suction Head

- The Total Head (***H_T***) represents the energy delivered by the pump when pumping any capacity or flow discharge, its value is calculated as follows,



$$H_T = h_d \pm h_s + l_d + l_s + \frac{v_d^{\,2}}{2g} \quad [3.8]$$

h_s is positive in Eq. 3.8 when the pump works with suction lift and negative with suction head

- The work done by a pump (***W_{hp}***) is also known as liquid horsepower,

$$W_{hp} = \frac{Q\,(gpm) \times H_T\,(ft) \times Sg}{3960} \quad [3.9]$$

- The Brake Horsepower (***B_{hp}***) is required to drive the pump,

$$B_{hp} = \frac{Q\,(gpm) \times H_T\,(ft) \times Sg}{3960 \times Eff_{pump}} \quad [3.10]$$

- From the two previous equations, the efficiency of a pump is represented by the ratio of the output from and input to the pump,

$$Eff_{pump} = \frac{W_{hp}}{B_{hp}} \quad [3.11]$$

- The Input Power to the motor is expressed in ***HP*** or ***KW*** as,

$$IP_{motor}\,(HP) = \frac{B_{hp}}{Eff_{motor}} \quad [3.12]$$

$$IP_{motor}\,(KW) = \frac{B_{hp} \times 0.746}{Eff_{motor}} \quad [3.13]$$

- The overall efficiency of the pump is affected by that of the motor,

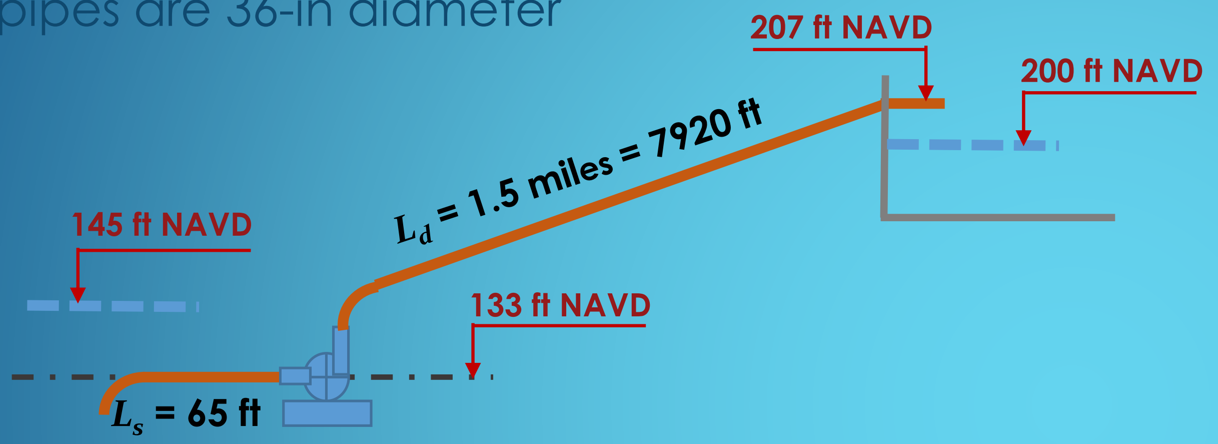
$$Eff_{p\&m} = Eff_{pump} \times Eff_{motor} \quad [3.14]$$

- The energy requirement (***KW-h***) per 1000 g of water pumped is calculated as follows,

$$KW - h/1000\,g = \frac{H_T(ft) \times 0.00315}{Eff_{p\&m}} \quad [3.15]$$

Pumps: Suction Lift and Suction Head

- 15) Calculate the power of the motor required to drive a pump for the depicted pumping system with free discharge into a tank. Also estimate the energy requirement for 8-hour operation per day. Both suction and discharge pipes are 36-in diameter



448.831 gpm/cfs

Q = 55.7 ft³/s = 25,000 gpm
D = 3.0 ft
f = 0.019
K = 0.55 *suction*
K = 0.40 *discharge*
Eff_{pump} = 83%
Eff_{motor} = 76%

Applying Eq. 3.8,

h_d = 207 ft – 133 ft = 74 ft h_s = 145 ft – 133 ft = 12 ft

$A = \pi$ (3 ft)^2 /4 = 7.07 ft^2 $v = Q/A$ = 7.88 ft/s $v^2/2g$ = **0.97 ft**

h_f = 0.019 x (7920 ft/3.0 ft)x 0.97 ft = **48.40 ft** (*discharge*)

h_f = 0.019 x (65 ft/3.0 ft)x 0.97 ft = **0.49 ft** (*suction*)

h_l = 0.40 x 0.97 ft = **0.39 ft** (*discharge*) h_l = 0.55 x 0.97 ft = **0.53 ft** (*suction*)

Pump problem

$$H_T = h_d \pm h_s + l_d + l_s + \frac{v_d^{\;2}}{2g} = 74\ ft - 12\ ft + 48.79\ ft + 0.93\ ft + 0.97\ ft = 112.69\ ft$$

From Eq. 3.10,

$$B_{hp} = \frac{Q\,(gpm) \times H_T\,(ft) \times Sg}{3960 \times Eff_{pump}} = \frac{25000 \times 112.69}{3960 \times 0.83} = 857.1\,HP$$

The Input Power of the motor is calculated from Eqs. 3.12 and 3.13,

$$IP_{motor}\,(HP) = \frac{B_{hp}}{Eff_{motor}} = \frac{857.1HP}{0.76} = 1127.8\,HP \quad \rightarrow \mathbf{1130\ HP}$$

$$IP_{motor}\,(KW) = \frac{B_{hp} \times 0.746}{Eff_{motor}} = 1127.8\,HP \times 0.746\,KW\,/\,HP = 841.3\,KW$$

Energy requirement is estimated from Eq. 3.15,

$$KW - h/1000\,g = \frac{H_T\,(ft) \times 0.00315}{Eff_{p\&m}} = \frac{112.69 \times 0.00315}{0.83 \times 0.76} = 0.563\,KW - h/1000\,g$$

Volume pumped (8-h/day) = 25000 gpm (8 hr) (60 min/h) = 12 x 10^6 g
Energy required = 0.563 KW–h/1000 g x 12,000 g = ***6753 KW–h***

Pump problem

Florida Department of Environmental Protection

Questions?

Thank you!