

# Florida Department of Environmental Protection



## Hydrology – 4b: Runoff and Flow Hydrographs

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- ▶ The **Unit Hydrograph** is a generalized way of defining the flood producing characteristics of a watershed
- ▶ A **Unit Hydrograph** is defined as a surface runoff versus time function which is generated from a unit depth of **Excess Rainfall** uniformly distributed over the watershed and occurring within a specified time duration
- ▶ It can be derived from a measured **Flood Hydrograph** knowing the equivalent uniform depth of the rainfall
- ▶ The following example illustrates the computation of the **Unit Hydrograph** from a recorded **Flood Hydrograph**

## The Unit Hydrograph

- **PROBLEM 4:** A **Flood Hydrograph** has been measured for a 18-hour flood in time intervals of 1.5 hours for a contributing watershed of 3-mile<sup>2</sup>. its peak flow was 660 ft<sup>3</sup>/sec at 7.05 hours. The **Baseflow** was estimated and  $T_B$  is 12 hours. Find the **Unit Hydrograph (DR)**

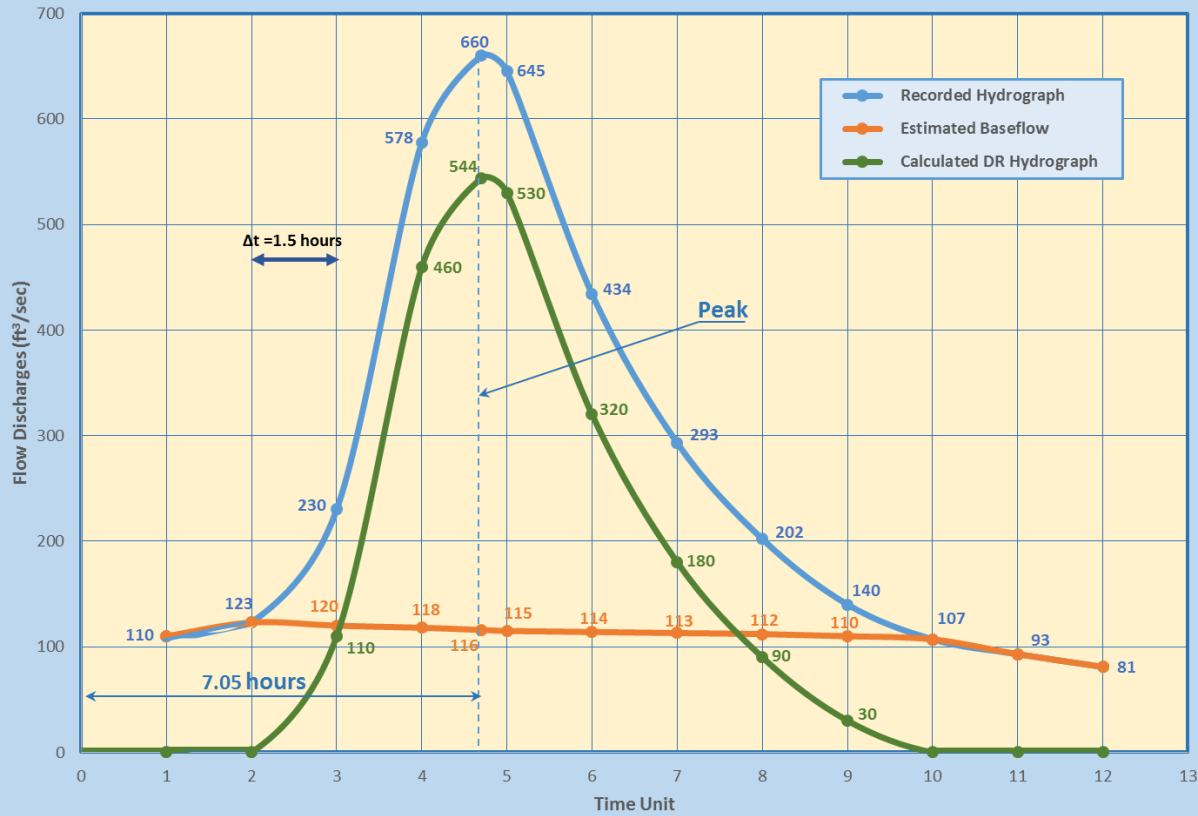
**Computation of Direct Runoff Hydrograph from Measured Flood Hydrograph and Estimated Baseflow**

| Time Unit                               | 1   | 2   | 3   | 4   | 4.7 | 5   | 6   | 7   | 8   | 9   | 10  | 11 | 12 |
|-----------------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|----|
| Flood Hydrog., Q (ft <sup>3</sup> /sec) | 110 | 123 | 230 | 578 | 660 | 645 | 434 | 293 | 202 | 140 | 107 | 93 | 81 |
| Baseflow (ft <sup>3</sup> /sec)         | 110 | 123 | 120 | 118 | 116 | 115 | 114 | 113 | 112 | 110 | 107 | 93 | 81 |
| DR Hydrograph (ft <sup>3</sup> /sec)    | 0   | 0   | 110 | 460 | 544 | 530 | 320 | 180 | 90  | 30  | 0   | 0  | 0  |

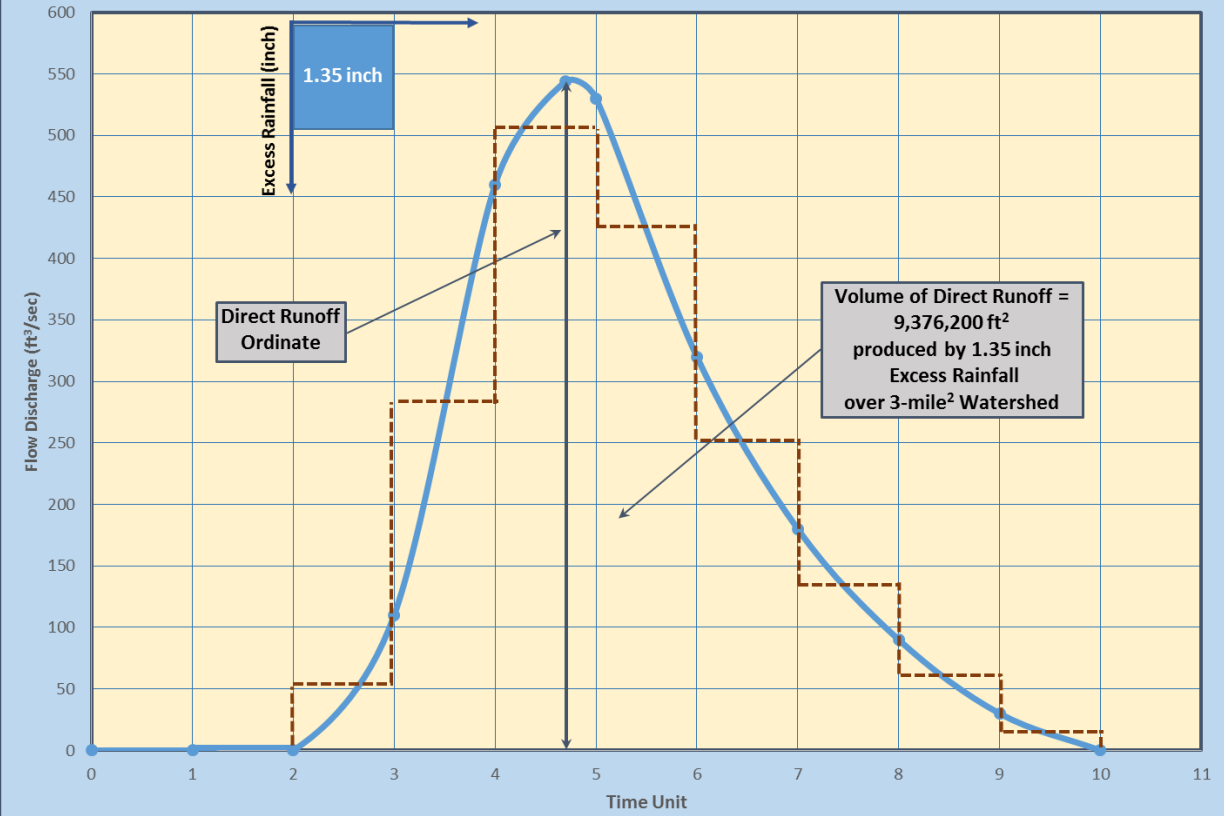
- The following figure depicts the hydrographs and baseflow from the above table

## The Unit Hydrograph

Recorded Hydrograph and Estimated Baseflow for Storm over 3-mile<sup>2</sup> Watershed



Direct Runoff Hydrograph (3-mile<sup>2</sup> Watershed and  $\Delta t = 1.5$  hours)



# The Unit Hydrograph

## Computation of Volume by integration of DR Hydrograph

= Excess Rainfall over the Watershed

| Time Unit | Direct Runoff Hydrograph (ft <sup>3</sup> /sec) | Time Interval $\Delta t$ | Average DR Hydrograph (ft <sup>3</sup> /sec) | Volume DR Hydrograph (ft <sup>3</sup> ) |
|-----------|-------------------------------------------------|--------------------------|----------------------------------------------|-----------------------------------------|
| 2         | 0                                               | 2 – 3                    | 55                                           | 297,000                                 |
| 3         | 110                                             | 3 – 4                    | 285                                          | 1,539,000                               |
| 4         | 460                                             | 4 – 5                    | 511                                          | 2,761,200                               |
| 4.7       | 544                                             | 5 – 6                    | 425                                          | 2,295,000                               |
| 5         | 530                                             | 6 – 7                    | 250                                          | 1,350,000                               |
| 6         | 320                                             | 7 – 8                    | 135                                          | 729,000                                 |
| 7         | 180                                             | 8 – 9                    | 60                                           | 324,000                                 |
| 8         | 90                                              | 9 – 10                   | 15                                           | 81,000                                  |
| 9         | 30                                              |                          |                                              |                                         |
| 10        | 0                                               |                          |                                              |                                         |
|           |                                                 | $\Sigma$                 | 1736                                         | 9,376,200                               |

Excess Rainfall Depth from the Storm over the Watershed

$$A_{\text{watershed}} = 3 \text{ mile}^2 \times (2.788 \times 10^7 \text{ ft}^2/\text{mile}^2) = 83.64 \times 10^6 \text{ ft}^2$$

$$\text{Depth}_{\text{ER}} = \frac{\text{Vol}_{\text{DR}}}{A_{\text{watershed}}} = \frac{9.3762 \times 10^6 \text{ ft}^3}{83.64 \times 10^6 \text{ ft}^2} \times (12 \text{ inch} / \text{ft}) = 1.35 \text{ inch}$$

The **Unit Hydrograph** can be developed by dividing all the ordinates of the **DR Hydrograph** by 1.35 inch. Therefore the volume of the **Unit Hydrograph** divided by the watershed area will yield and **Excess Rainfall** depth of 1-inch

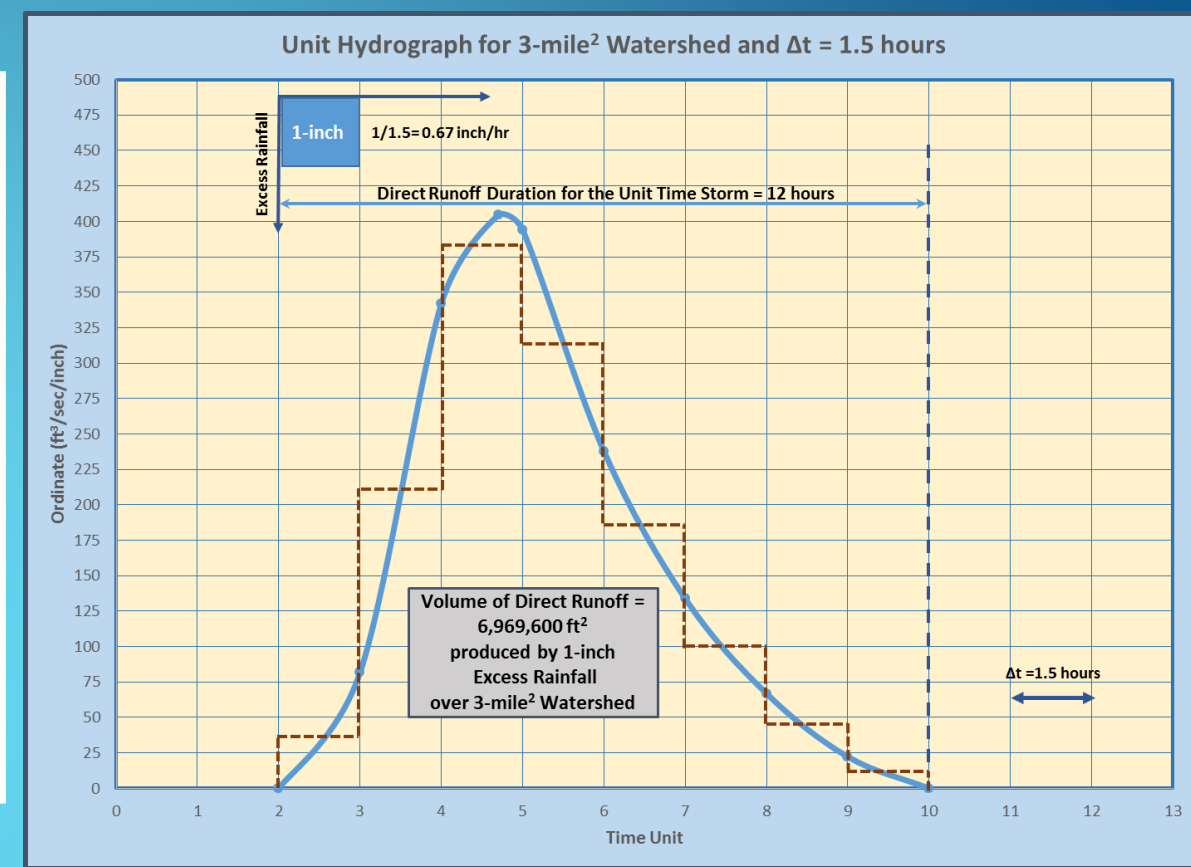
# The Unit Hydrograph

## Computation of the Unit Hydrograph and Verification

| Time Unit | Direct Runoff Hydrograph (ft <sup>3</sup> /sec) | Unit Hydrograph (ft <sup>3</sup> /sec) | Time Interval Δt | Average U Hydrograph (ft <sup>3</sup> /sec) | Volume U Hydrograph (ft <sup>3</sup> ) |
|-----------|-------------------------------------------------|----------------------------------------|------------------|---------------------------------------------|----------------------------------------|
| 2         | 0                                               | 0.0                                    | 2 – 3            | 40.9                                        | 220,768.7                              |
| 3         | 110                                             | 81.8                                   | 3 – 4            | 211.8                                       | 1,143,983.1                            |
| 4         | 460                                             | 341.9                                  | 4 – 5            | 380.1                                       | 2,052,479.6                            |
| 4.7       | 544                                             | 404.4                                  | 5 – 6            | 315.9                                       | 1,705,939.7                            |
| 5         | 530                                             | 394.0                                  | 6 – 7            | 185.8                                       | 1,003,494.0                            |
| 6         | 320                                             | 237.9                                  | 7 – 8            | 100.3                                       | 541,886.7                              |
| 7         | 180                                             | 133.8                                  | 8 – 9            | 44.6                                        | 240,838.5                              |
| 8         | 90                                              | 66.9                                   | 9 – 10           | 11.1                                        | 60,209.6                               |
| 9         | 30                                              | 22.3                                   |                  |                                             |                                        |
| 10        | 0                                               | 0.0                                    |                  |                                             |                                        |
|           |                                                 |                                        | Σ                | 1290.7                                      | 6,969,600.0                            |

$$Depth_{ER} = \frac{Vol_U}{A_{watershed}} = \frac{6.9696 \times 10^6 \text{ ft}^3}{83.64 \times 10^6 \text{ ft}^2} \times (12 \text{ inch} / \text{ft}) = 1.0 \text{ inch}$$

## The Unit Hydrograph



- ▶ As frequently happens when several periods of **Excess Rainfall** occur in sequence, the resulting surface **DR Hydrograph** is the result of the superposition of a series of hydrographs each initiated by its own respective occurrence of **Excess Rainfall**
- ▶ The problem consists of decomposing the measured composite **Flood Hydrograph** into its component hydrographs and then of deriving the **Unit Hydrograph**
- ▶ The following equation represents the successive ordinates of the recorded surface **DR Hydrograph**

## The Unit Hydrograph

$$Q_k = U_1 \times ER_i + U_2 \times ER_{i-1} + U_3 \times ER_{i-2} + \dots U_n \times ER_{i-n+1} \dots [4.9]$$

where  $Q_k$  is the  $k^{th}$  ordinate of the **DR Hydrograph**,  $U_n$  is the  $n^{th}$  ordinate of the **Unit Hydrograph** and  $ER_i$  is the  $i^{th}$  value of **Excess Rainfall**

- The **Unit Hydrograph** solution has the following form and can be solved by a Matrix Method

$$Q_1 = ER_1 \times U_1$$

$$Q_2 = ER_1 \times U_2 + ER_2 \times U_1$$

$$Q_3 = ER_1 \times U_3 + ER_2 \times U_2 + ER_3 \times U_1$$

$$Q_4 = ER_1 \times U_4 + ER_2 \times U_3 + ER_3 \times U_2 + ER_4 \times U_1$$

And in general matrix form is:  $Q = ER \cdot U \dots [4.10]$

## The Unit Hydrograph

- The **Excess Rainfall** is a square (lower triangular) matrix, the **Unit Hydrograph** and the surface **DR Hydrograph** are both column vectors, as follows

$$\begin{bmatrix} ER_1 & 0 & 0 & 0 & \cdot & \cdot & 0 \\ ER_2 & ER_1 & 0 & 0 & & & 0 \\ ER_3 & ER_2 & ER_1 & 0 & & & 0 \\ \cdot & & & & & & \\ ER_i & ER_{i-1} & & & & & ER_1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ \cdot \\ U_j \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ \cdot \\ Q_k \end{bmatrix} \quad \text{with } k = i+j-1 \dots [4.11]$$

when the surface **DR Hydrograph** is known, the matrix solution become Eq. 4.10

- A numerical solution is expressed as a transformation of the **Excess Rainfall**, which is an inexact duplication of the **DR Hydrograph**

## The Unit Hydrograph

$$U = (ER^T \ ER)^{-1} \ ER \cdot Q \dots [4.12]$$

where  $ER^T$  is the transpose matrix of the **Excess Rainfall** and  $(ER^T \ ER)^{-1}$  represents the inverse matrix of the product of the **Excess Rainfall** transpose matrix and its matrix

- ▶ By adjusting the **Unit Hydrograph**, the duplication effect may be improved. An optimization scheme can be applied to the **Unit Hydrograph**
- ▶ There are many numerical schemes that facilitate these computations even available in standard calculators
- ▶ The following example illustrate an approximate solution to the **Unit Hydrograph** based on the simultaneous equations derived from Eq. 4.10

## The Unit Hydrograph

- **PROBLEM 5:** A surface **DR Hydrograph** has been estimated from a **Flood Hydrograph** (after subtracting the **Baseflow**) as well as the **Excess Rainfall** from the **Hyetograph**. Their values are shown in the following table. The time interval is 1-hour. Compute the **Unit Hydrograph** for the storm

**Estimated Direct Runoff Hydrograph (from Flood Hydrograph) and excess rainfall (from Hyetograph)**

| Time Unit                               | 1    | 2   | 3    | 4   | 5   | 6   | 7   | 8  | 9  | 10 |
|-----------------------------------------|------|-----|------|-----|-----|-----|-----|----|----|----|
| DR Hydrograph, Q (ft <sup>3</sup> /sec) | 66   | 390 | 678  | 419 | 572 | 318 | 173 | 94 | 24 | 5  |
| Excess Rainfall (inch)                  | 1.27 | 0   | 0.53 |     |     |     |     |    |    |    |

- From Eq. 4.12,  $j$  (index of Unit Hydrograph ordinates) =  $k - i + 1 = 10 - 3 + 1 = 8$   
It means that the **Unit Hydrograph** will have 8 ordinates

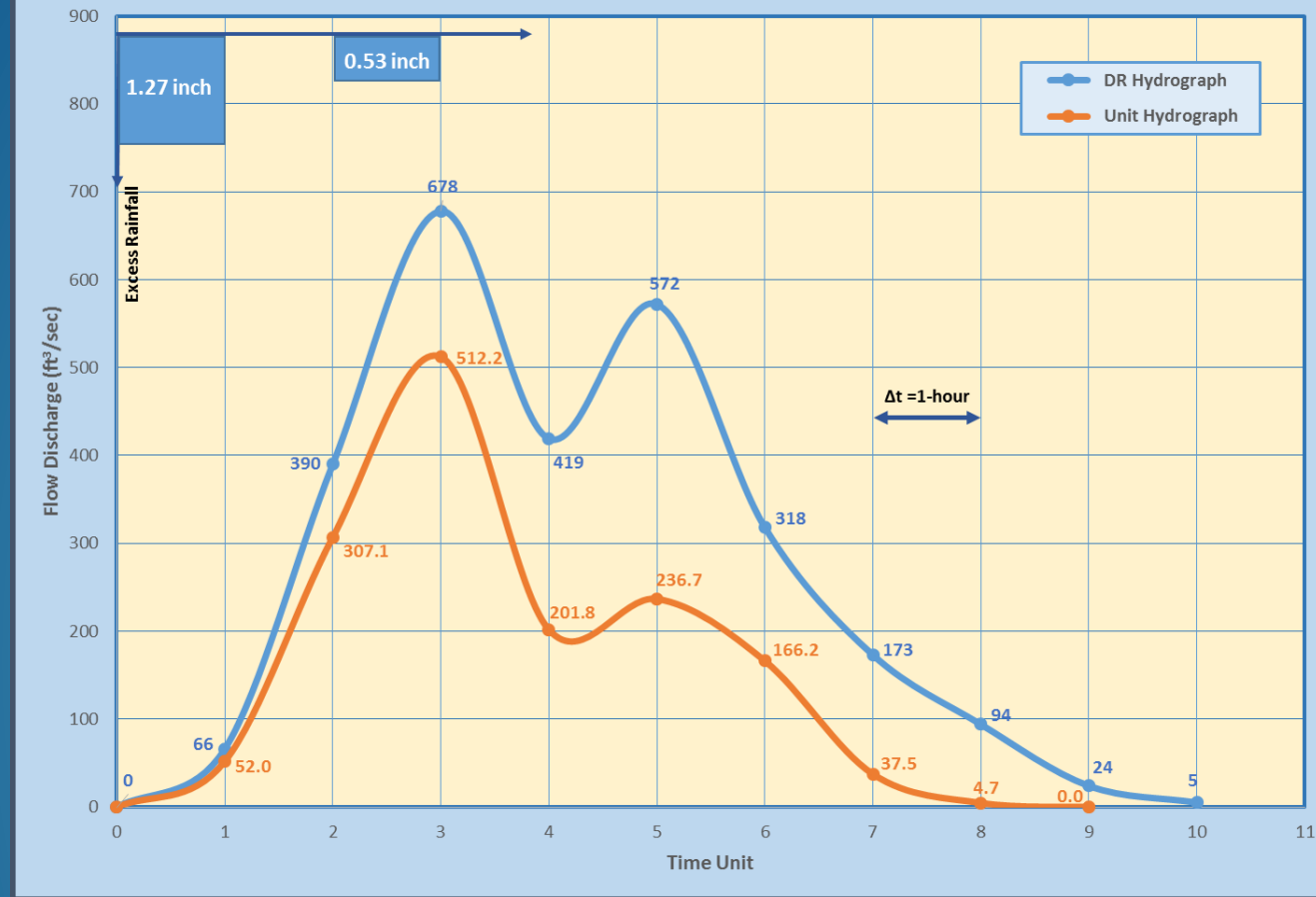
## The Unit Hydrograph

## Calculation of Unit Hydrograph (Consecutive Excess Rainfall Periods from Storm Hyetograph)

| Excess Rainfall | 4.7 | (inch)                 | 1.27          | 0              | 0.53           | Equation to Compute $U_j$              | $U_j$ | U (ft <sup>3</sup> /sec/inch) |
|-----------------|-----|------------------------|---------------|----------------|----------------|----------------------------------------|-------|-------------------------------|
| $Q_k$           |     | (ft <sup>3</sup> /sec) | $ER_1 U_i$    | $ER_2 U_{i-1}$ | $ER_3 U_{i-2}$ |                                        |       |                               |
| $Q_0$           |     | 0                      |               |                |                |                                        | $U_0$ | 0.0                           |
| $Q_1$           |     | 66                     | $ER_1 U_1$    | $ER_2 U_0$     | $ER_3 U_0$     | $Q_1 = ER_1 U_1 + ER_2 U_0 + ER_3 U_0$ | $U_1$ | 52.0                          |
| $Q_2$           |     | 390                    | $ER_1 U_2$    | $ER_2 U_1$     | $ER_3 U_0$     | $Q_2 = ER_1 U_2 + ER_2 U_1 + ER_3 U_0$ | $U_2$ | 307.1                         |
| $Q_3$           |     | 678                    | $ER_1 U_3$    | $ER_2 U_2$     | $ER_3 U_1$     | $Q_3 = ER_1 U_3 + ER_2 U_2 + ER_3 U_1$ | $U_3$ | 512.2                         |
| $Q_4$           |     | 419                    | $ER_1 U_4$    | $ER_2 U_3$     | $ER_3 U_2$     | $Q_4 = ER_1 U_4 + ER_2 U_3 + ER_3 U_2$ | $U_4$ | 201.8                         |
| $Q_5$           |     | 572                    | $ER_1 U_5$    | $ER_2 U_4$     | $ER_3 U_3$     | $Q_5 = ER_1 U_5 + ER_2 U_4 + ER_3 U_3$ | $U_5$ | 236.7                         |
| $Q_6$           |     | 318                    | $ER_1 U_6$    | $ER_2 U_5$     | $ER_3 U_4$     | $Q_6 = ER_1 U_6 + ER_2 U_5 + ER_3 U_4$ | $U_6$ | 166.2                         |
| $Q_7$           |     | 173                    | $ER_1 U_7$    | $ER_2 U_6$     | $ER_3 U_5$     | $Q_7 = ER_1 U_7 + ER_2 U_6 + ER_3 U_5$ | $U_7$ | 37.5                          |
| $Q_8$           |     | 94                     | $ER_1 U_8$    | $ER_2 U_7$     | $ER_3 U_6$     | $Q_8 = ER_1 U_8 + ER_2 U_7 + ER_3 U_6$ | $U_8$ | 4.7                           |
| $Q_9$           |     | 24                     | $ER_1 U_9$    | $ER_2 U_8$     | $ER_3 U_7$     |                                        | $U_9$ | 0.0                           |
| $Q_{10}$        |     | 5                      | $ER_1 U_{10}$ | $ER_2 U_9$     | $ER_3 U_8$     |                                        |       |                               |
| $Q_{11}$        |     | 0                      | $ER_1 U_{11}$ | $ER_2 U_{10}$  | $ER_3 U_9$     |                                        |       |                               |

# The Unit Hydrograph

Unit Hydrograph for two Excess Rainfall Periods (1.80 inch)

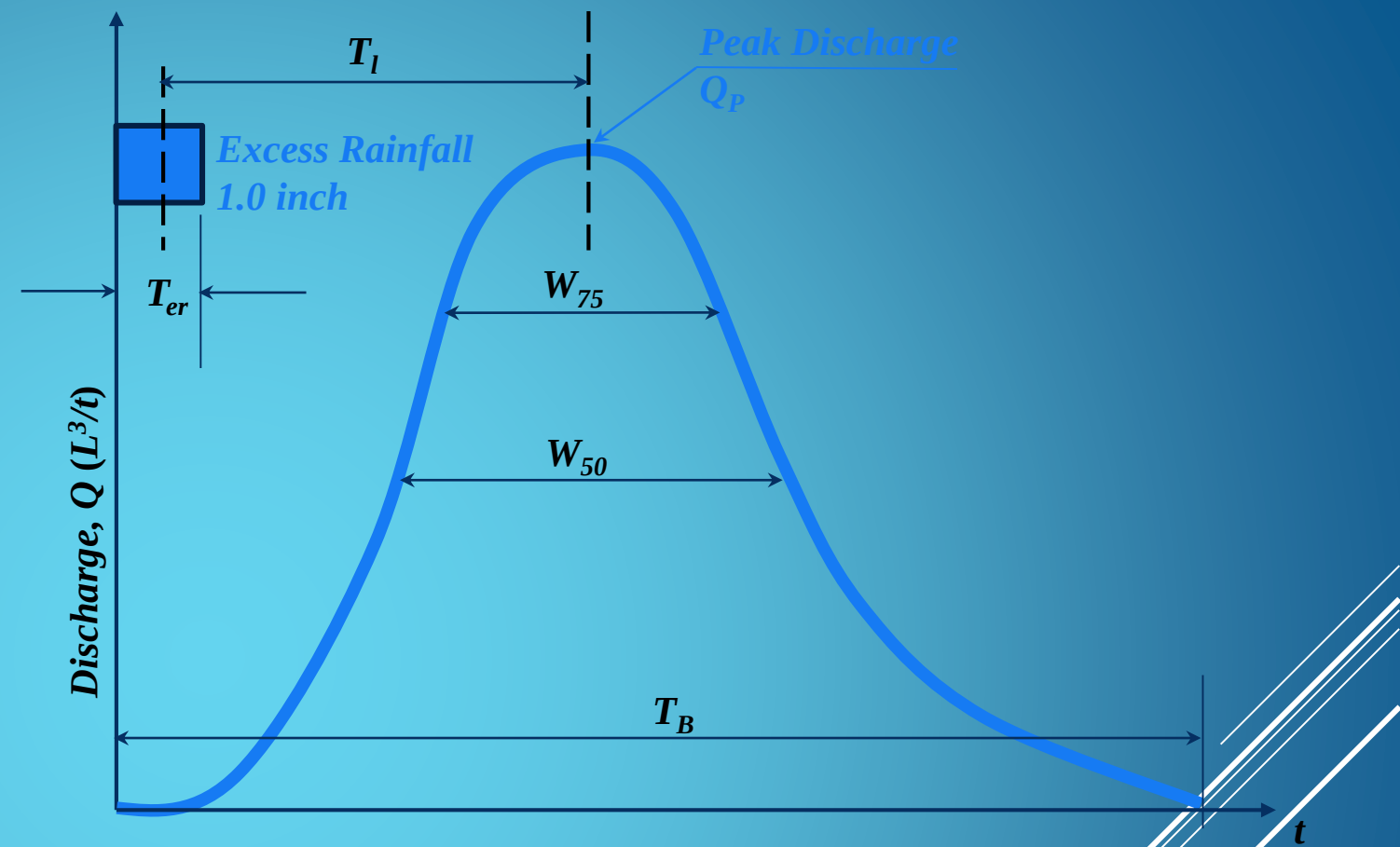


# The Unit Hydrograph

- ▶ The **Synthetic Unit Hydrograph** is a procedure developed to allow the application of the **Unit Hydrograph** concept in a design problem where direct measurements of a **Flood Hydrograph** are missing
- ▶ The linear characteristics of **Unit Hydrographs** for a watershed as they are generated by superposition of successive excess rainfall impulses are a definite advantage in constructing more complex storm discharge hydrographs
- ▶ Synthetic techniques (in some cases applicable to specific sites and locations) have evolved continuously providing significant tools to analyzed a wide range of water resources projects

## Synthetic Unit Hydrographs (SUH)

- The **Snyder's Method** has been widely employed by the US COE. Even though this method does not provide a complete time distribution of the discharge hydrograph, it allows computation of **Lag Time**, **Excess Rainfall** and **Base Time** durations, **Peak Discharge**, and hydrograph time widths at 50 and 75% of the **Peak Discharge**



*SUH*: Snyder's Method

- Snyder developed a **Lag time**,  $T_l$  relationship for watersheds ranging from 10 to 10,000 miles<sup>2</sup> located in the Appalachian Highlands as follows,

$$T_l = C_t \times (L \times L_{ca})^{0.3} \dots [4.13]$$

where  $T_l$  is the **Lag Time** (hr),  $C_t$  is a coefficient for the slope and storage of the watershed,  $L$  is the length of the main stream channel (miles) and  $L_{ca}$  is the length along the main channel to a point nearest the watershed centroid (miles).  $C_t$  accounts for a wide variation in topography, from plains to mountains with values ranging from 1.8 to 2.2 with an average of 2.0 indicating no major changes for similar study areas

## *SUH*: Snyder's Method

Further analysis showed much larger variation with slopes. Steeper slopes yielded lower values of  $C_t$  as 0.4 in southern California and flatter slopes did higher values of  $C_t$  as 8.0 along the Gulf of Mexico

- ▶ The following equation relates the duration of **Excess Rainfall** as a function of **Lag Time**

$$T_{er} = \frac{T_l}{5.5} \dots [4.14]$$

where  $T_{er}$  is the duration of the unit **Excess Rainfall** (hr)

- ▶ This method defines initially a value of  $T_l$  and its associated value of  $T_{er}$  from Eq. 4.14. If the duration of the unit **Excess Rainfall** has to be changed, the **Lag Time**  $T_l$  will change as well, and it will have to be recalculated by the following equation

## *SUH*: Snyder's Method

$$T_L = T_l + 0.25 \times (T_{ER} - T_{er}) \dots [4.15]$$

where  $T_L$  is the adjusted **Lag Time** (hr) and  $T_{ER}$  is the desired duration of **Excess Rainfall** that replaces  $T_{er}$  (hr)

- The **Peak Discharge** is computed as follows

$$Q_p = \frac{640 \times C_p \times A}{T_l} \dots [4.16]$$

where  $Q_p$  is the **Peak Discharge** (ft<sup>3</sup>/sec),  $C_p$  is the peak coefficient varying from 0.56 to 0.69 with an average value of 0.60 and  $A$  is the area of the watershed (miles<sup>2</sup>)

- The **Base Time**,  $T_B$  of the **SUH** by the Snyder's Method is

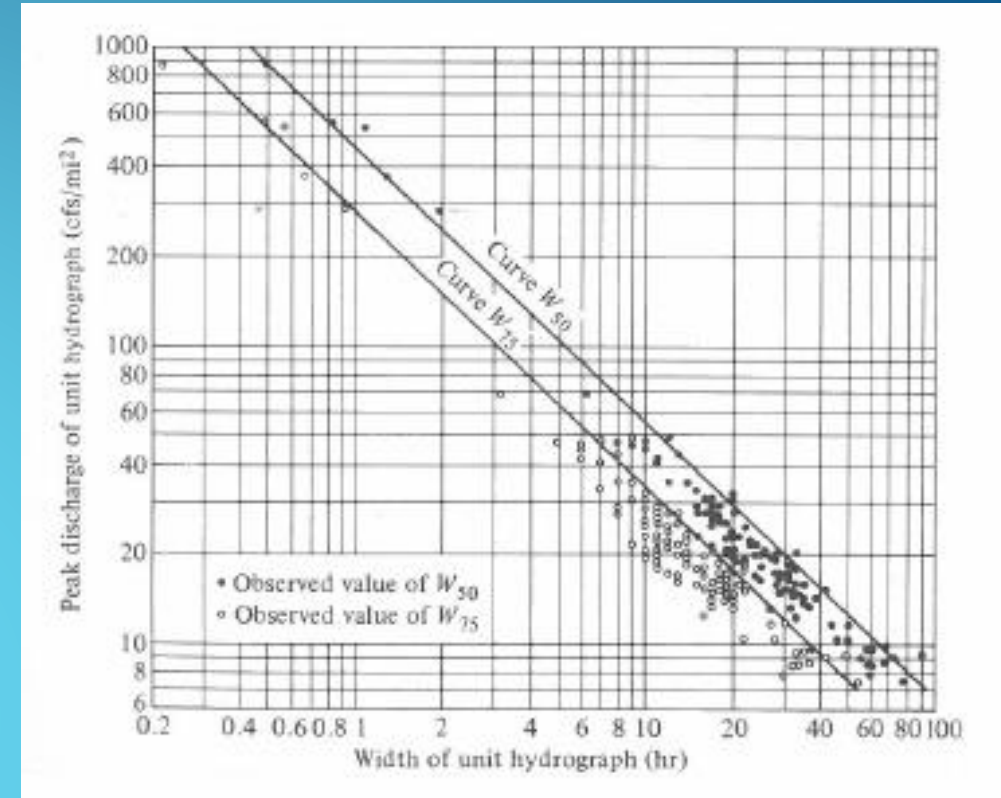
## *SUH*: Snyder's Method

$$T_B = 3 + \frac{T_l}{8} \dots [4.17]$$

where  $T_B$  is the **Base Time** of the **SUH** (days) and  $T_l$  is the **Lag Time** (hr)

- Once all computations from Eqs. 4.13 to 4.17 have been done, it is necessary to estimate  $W_{50}$  and  $W_{75}$ . These two time widths are obtained from the graph with the Peak Discharge of Unit Hydrograph as

$$q_P = \frac{Q_P}{A} \dots [4.18]$$



As a rule of thumb, the time width at  $W_{50}$  and  $W_{75}$  ordinates should be distributed at 1/3 and 2/3 to the left and right-hand sides of the **Snyder's SUH**, respectively

# SUH: Snyder's Method

- **PROBLEM 6:** An analysis on the depicted 1500-mi<sup>2</sup> watershed has given the shown results. The duration of the **Excess Rainfall** ( $T_{ER}$ ) was 6.0 hr. Plot the **SUH** using the Snyder's Method for average conditions of slope and storage of the watershed as well as of its discharge

For average conditions:  $C_t = 2$  and  $C_p = 0.60$

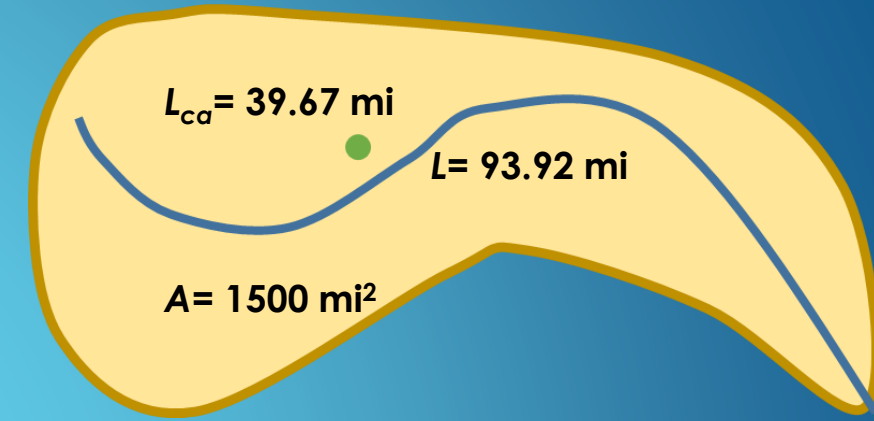
From Eq. 4.13  $\rightarrow T_L = 2 \times (93.92 \times 39.67)^{0.3} = 23.57$  hr

From Eq. 4.14  $\rightarrow T_{er} = 23.57 \text{ hr} / 5.5 = 4.29$  hr

It is necessary to recalculate  $T_L$  for the desired duration of 6.0 hr for the **Excess Rainfall** ( $T_{ER}$ ), so

From Eq. 4.15  $\rightarrow T_L = 23.57 \text{ hr} + 0.25 \times (6 - 4.29) \text{ hr} = 24$  hr

From Eq. 4.16  $\rightarrow Q_p = 640 \times 0.60 \times 1500 / 24 = 24,000 \text{ ft}^3/\text{sec}$



## *SUH*: Snyder's Method

From Eq. 4.17  $\rightarrow T_B = 3 + 24/8 = 6 \text{ days} = 144 \text{ hr}$

From Eq. 4.18  $\rightarrow q_P = 24,000 \text{ ft}^3/\text{sec}/1500 \text{ mi}^2 = 16 \text{ cfs}/\text{mi}^2$

With  $q_P = 16 \text{ cfs}/\text{mi}^2$ , the time widths are obtained from the graph

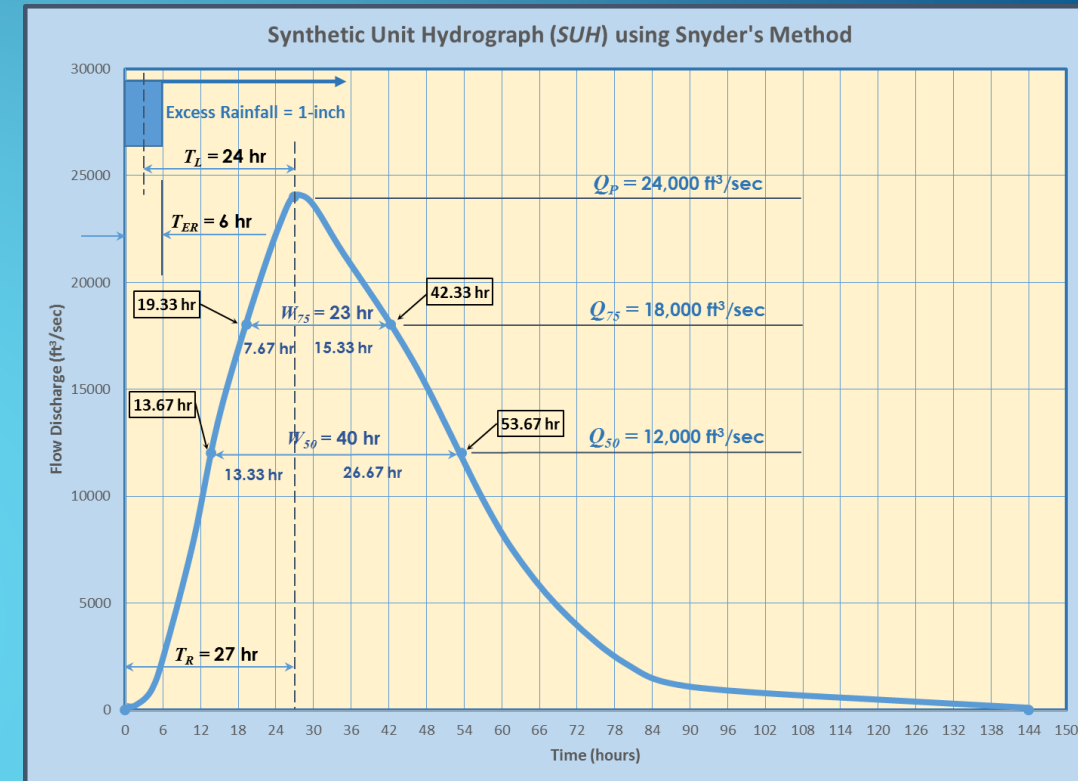
$W_{50} = 40 \text{ hr}$  and  $W_{75} = 23 \text{ hr}$

**Time to Rise ( $T_R$ )** =  $24 \text{ hr} + 6 \text{ hr} / 2 = 27 \text{ hr} \rightarrow Q_P = 24,000 \text{ cfs}$

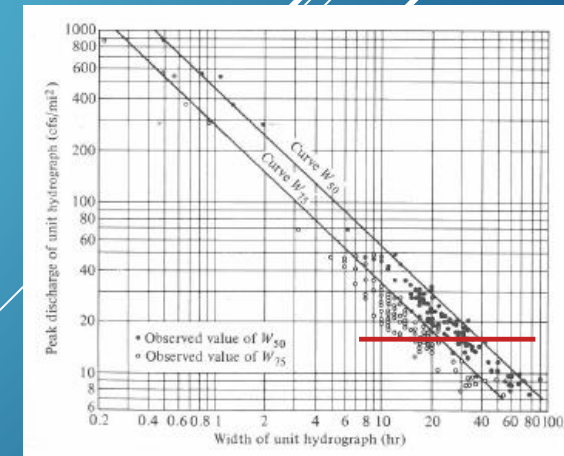
$Q_{50} = 12,000 \text{ cfs} \rightarrow T_{\text{left}} = 27 \text{ hr} - 40 \text{ hr} \times (1/3) = 13.67 \text{ hr}$   
 $T_{\text{right}} = 27 \text{ hr} + 40 \text{ hr} \times (2/3) = 53.67 \text{ hr}$

$Q_{75} = 18,000 \text{ cfs} \rightarrow T_{\text{left}} = 27 \text{ hr} - 23 \text{ hr} \times (1/3) = 19.33 \text{ hr}$   
 $T_{\text{right}} = 27 \text{ hr} + 23 \text{ hr} \times (2/3) = 42.33 \text{ hr}$

**Base Time ( $T_B$ )** =  $144 \text{ hr} \rightarrow Q = 0$



## SUH: Snyder's Method



- ▶ The Soil Conservation Service (**SCS**) method for constructing **SUH** was developed based on dimensionless hydrographs analyzed of a large range of watersheds in size and geographical locations
- ▶ This method requires only the determination of  $T_R$  (**Time to Rise**) and  $Q_p$  (**Peak Discharge**)
- ▶ The **Lag Time** is calculated by the following equation

$$T_l = \frac{L^{0.8} \times (S + 1)^{0.7}}{1900 \times \sqrt{Y}} \dots [4.19]$$

where  $L$  is the length of the main stream channel (ft),  $S$  is the potential maximum retention (inch) and  $Y$  is the average watershed slope (%)

## **SUH: SCS Dimensionless Method**

- ▶ The potential maximum retention is calculated by the SCS proposed equation

$$S = \frac{1000}{CN} - 10 \dots [4.20]$$

where **CN** is the **Curve Number** a defined value based on the soil type of the watershed

- ▶ The **Peak Discharge** is computed as follows

$$Q_P = \frac{484 \times A}{T_R} \dots [4.21]$$

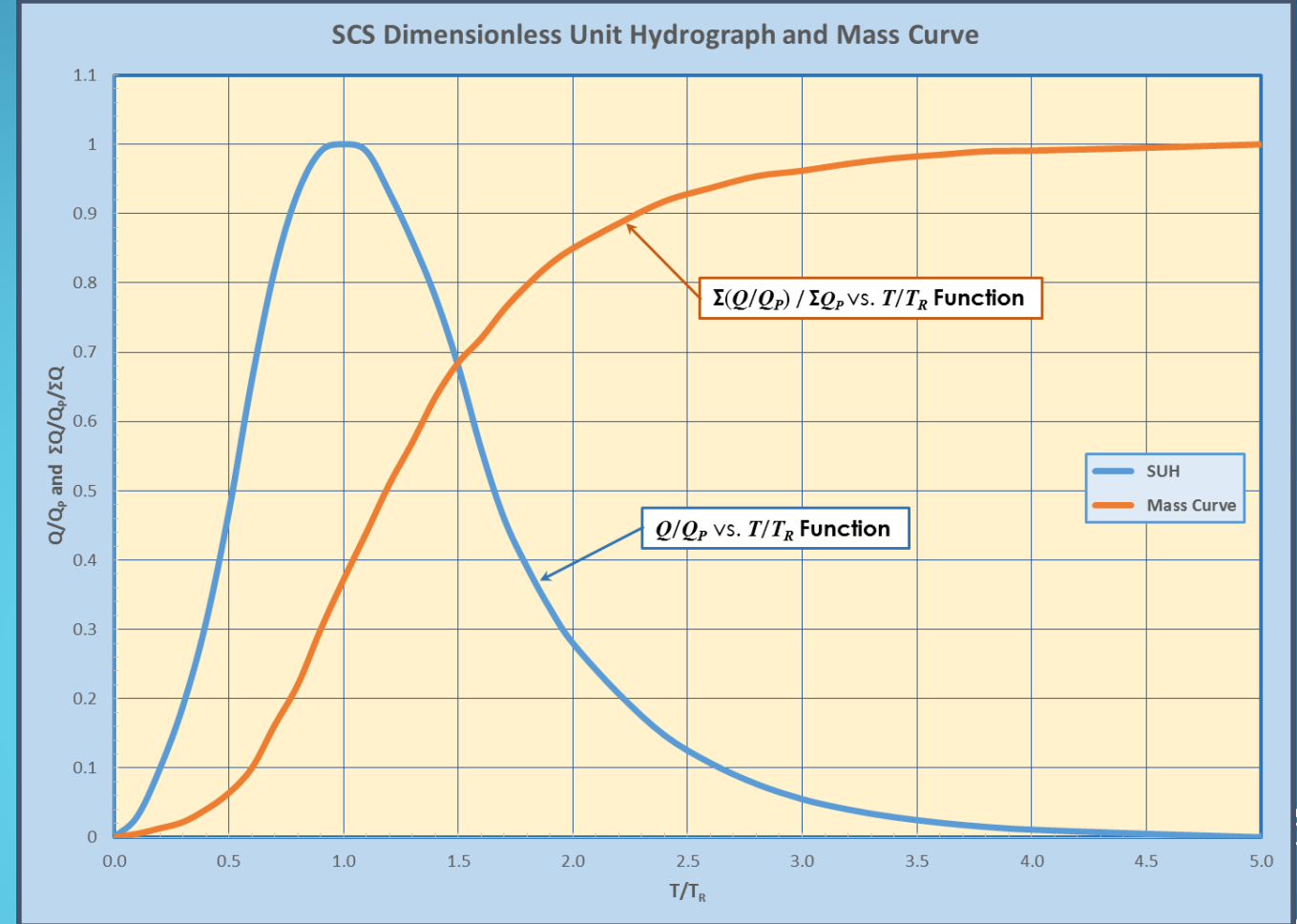
where  $Q_P$  is the **Peak Discharge** (ft<sup>3</sup>/sec), **A** is the area of the watershed (miles<sup>2</sup>), and  $T_R$  is the **Time to Rise**, calculated by the following equation

## *SUH*: SCS Dimensionless Method

$$T_R = \frac{D}{2} + T_l \dots [4.22]$$

where ***D*** is the duration of the **Excess Rainfall** (hr) and ***T<sub>R</sub>*** is **Time to Rise** (hr)

- ▶ The adjacent graph depicts the values to calculate the time and discharge ordinates after computation of ***Q<sub>P</sub>*** and ***T<sub>R</sub>*** for the ***SUH*** developed by the ***SCS***



# ***SUH***: SCS Dimensionless Method

- **PROBLEM 7:** A woodland is in “fair hydrologic conditions” with soils of the C-type (Hydrologic Soil Group) and has drainage area of 70 mi<sup>2</sup>. Its main stream channel is 15.5 mi and the average watershed slope has been estimated as 3.12%. Compute the **SUH** of **D** = 4 hour from the SCS Dimensionless Method

For the given characteristics of the watershed and its soils, **CN** = 73

From Eq. 4.20 → **S** = 1000/73 + 10 = **3.70 inch**

From Eq. 4.19 →  $T_l = \frac{81,840^{0.8} \text{ ft} \times (3.7 \text{ inch} + 1)^{0.7}}{1900 \times \sqrt{3.12}} = 7.5 \text{ hr}$

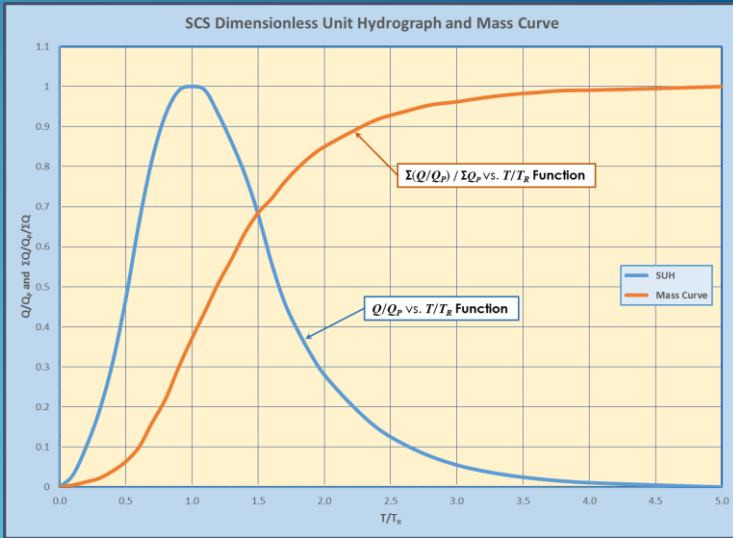
From Eq. 4.22 →  $T_R = \frac{4 \text{ hr}}{2} + 7.5 \text{ hr} = 9.5 \text{ hr}$

## *SUH*: SCS Dimensionless Method

$$\text{From Eq. 4.21} \rightarrow Q_P = \frac{484 \times 70 \text{ mi}}{9.5 \text{ hr}} = 3566.3 \text{ ft}^3 / \text{sec}$$

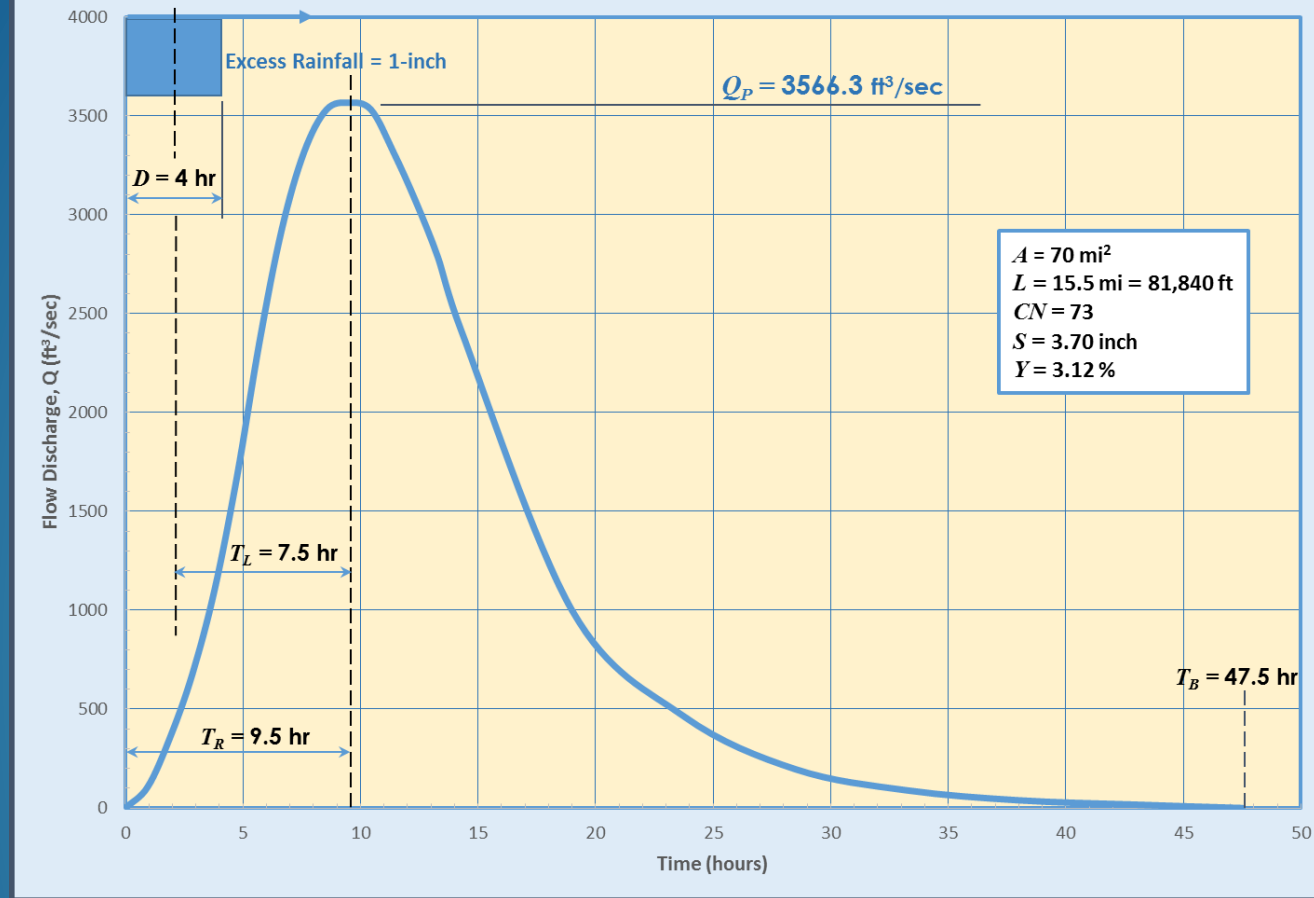
# Computation of the ordinates of the SUH from SCS Method (Dimensionless)

| $T/T_R$<br>$T_R = 9.5 \text{ hr}$ | $Q/Q_P$<br>$Q_P = 3566.3 \text{ cfs}$ | $T$<br>(hr) | $Q$<br>(cfs) | $T/T_R$<br>$T_R = 9.5 \text{ hr}$ | $Q/Q_P$<br>$Q_P = 3566.3 \text{ cfs}$ | $T$<br>(hr) | $Q$<br>(cfs) |
|-----------------------------------|---------------------------------------|-------------|--------------|-----------------------------------|---------------------------------------|-------------|--------------|
| 0.0                               | 0.0                                   | 0.00        | 0.0          | 1.5                               | 0.68                                  | 14.25       | 2425.1       |
| 0.1                               | 0.03                                  | 0.95        | 107.0        | 2.0                               | 0.28                                  | 19.00       | 998.6        |
| 0.2                               | 0.1                                   | 1.90        | 356.6        | 2.5                               | 0.13                                  | 23.75       | 463.3        |
| 0.5                               | 0.47                                  | 4.75        | 1676.2       | 3.0                               | 0.055                                 | 28.50       | 196.1        |
| 0.7                               | 0.82                                  | 6.65        | 2924.4       | 3.5                               | 0.025                                 | 33.25       | 89.2         |
| 0.8                               | 0.93                                  | 7.60        | 3316.7       | 4.0                               | 0.011                                 | 38.00       | 39.2         |
| 1.0                               | 1.0                                   | 9.50        | 3566.3       | 4.5                               | 0.005                                 | 42.75       | 17.8         |
| 1.2                               | 0.93                                  | 11.40       | 3316.7       | 5.0                               | 0.0                                   | 47.50       | 0.0          |



# SUH: SCS Dimensionless Method

Synthetic Unit Hydrograph using SCS Method (from Dimensionless Hydrograph)



*SUH*: SCS Dimensionless Method

- ▶ This method of generating ***SUH*** was developed initially for midwestern watersheds by D.M. Gray in 1961
- ▶ Gray's Method is based on dimensionalizing the incomplete Gamma Distribution and analyzing results in dimensionless form
- ▶ An approximate upper limit of watershed area for application of this method to the geographic areas of Central Iowa, Missouri, Illinois and Wisconsin is 100 mi<sup>2</sup>
- ▶ Watershed and Storage Factors are determined by the following equations

*SUH*: Gray's Method

$$WF = \frac{L}{\sqrt{S}} \dots [4.23]$$

$$SF = \frac{T_R}{\gamma'} \dots [4.24]$$

$$SF = a \times WF^b \dots [4.25]$$

where **WF** is the **Watershed Factor**, **L** and **S** are the length and slope of the stream channel, respectively (mi and %), **SF** is the **Storage Factor**, **T<sub>R</sub>** is the **Time to Rise** (min), and **a** and **b** are site-specific regression factors

- The **Storage Factor** is related to **Time to Rise** as follows

$$SF = \frac{1}{\frac{2.676}{T_R} + 0.0139} \dots [4.26]$$

Once **T<sub>R</sub>** has been calculated the flow discharge can be calculated for any value of the fraction **T/T<sub>R</sub>**

## SUH: Gray's Method

$$Q_{T/T_R} = \frac{25 \times (\gamma')^q}{\Gamma(q)} \times (e^{-\gamma' \times T/T_R}) \times \left( \frac{T}{T_R} \right)^{q-1} \dots [4.27]$$

where  $Q_{T/T_R}$  is the dimensionless discharge for  $T/T_R$  (%),  $q$  is a **Shape Parameter** =  $1 + \gamma'$ , and  $\Gamma$  is the **Gamma Function** =  $(q - 1)!$  and  $e$  is the base of the natural logarithms

- ▶ The volume of the unit hydrograph is calculated as follows, considering an **Excess Rainfall** of 1.0 inch

$$V = 1.0 \text{ inch} \times A \text{ mi}^2 \times 640 \frac{\text{acre}}{\text{mi}^2} \times 0.0833 \frac{\text{ft}}{\text{inch}} \times 43,560 \frac{\text{ft}^2}{\text{acre}} \dots [4.28]$$

- ▶ The above volume,  $V$  ( $\text{ft}^3$ ) has to match the volume for the dimensionless hydrograph, which is

## SUH: Gray's Method

$$V_D = \sum cfs \times 0.25 \times T_R \times 60 \frac{\text{sec}}{\text{min}} \dots [4.29]$$

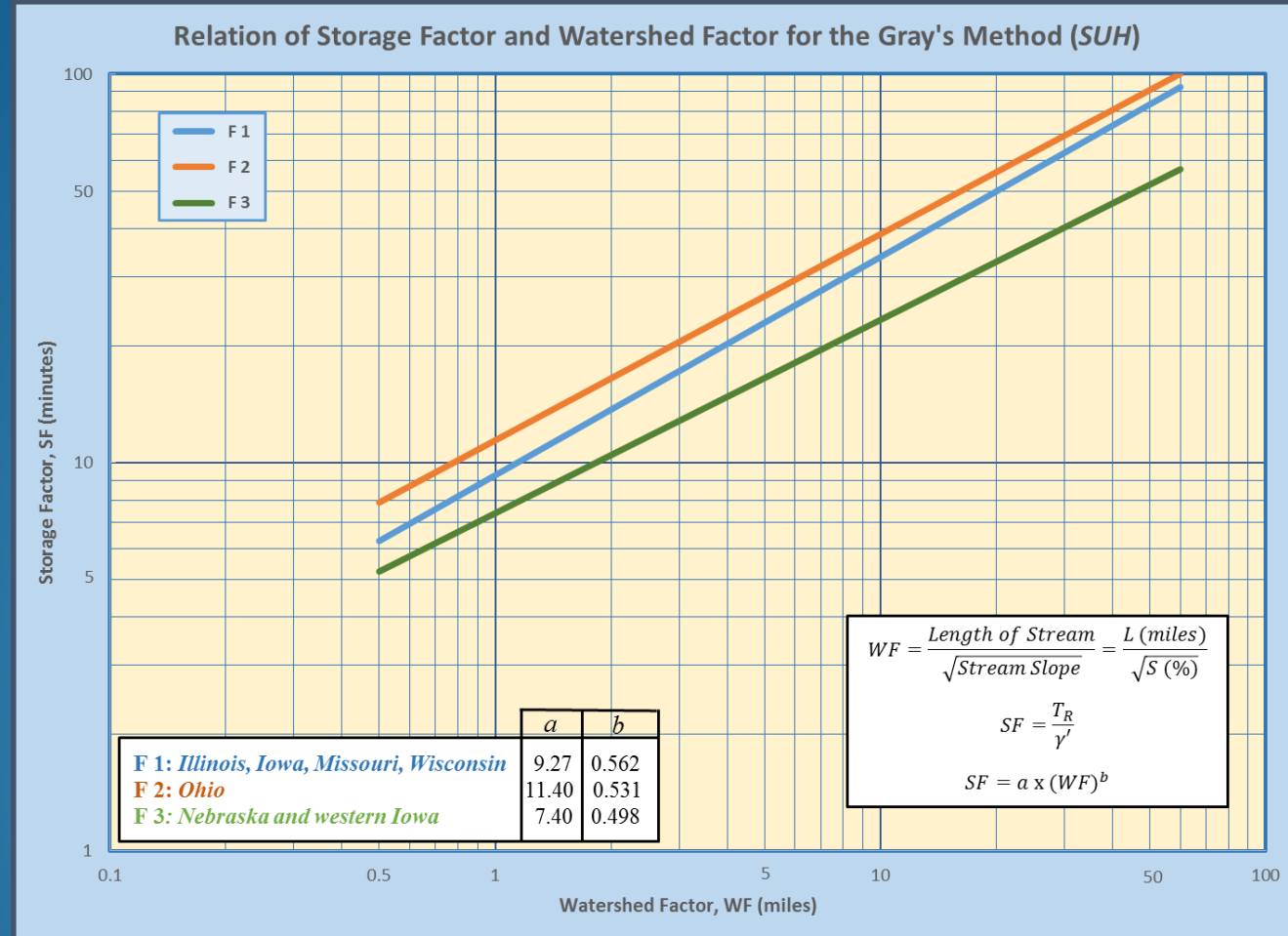
by equating  $V$  and  $V_D$  (Eqs. 4.28 and 4.29), the term  $\sum cfs$  is calculated

- ▶ The Unit Hydrograph ordinates are calculated by the following equation

$$Q_u = \frac{Q_{T/T_R}}{100} \times \sum cfs \dots [4.30]$$

- ▶  $T$  of the dimensionless **Hydrograph** is converted to absolute time by multiplying  $T/T_R \times T_R$  for each flow discharge ( $Q_u$ ). It is important to realize that **Runoff** does not begin until the centroid of rainfall has been reached, or  $T_R/8$

## SUH: Gray's Method



SUH: Gray's Method

- **PROBLEM 8:** A watershed has a drainage area of 0.62 mi<sup>2</sup>, its stream channel is 0.98 mi long with an average slope of 1.45%. Its storage factor coincides with those from F 1 (i.e.,  $a = 9.27$  and  $b = 0.562$ ). Apply Gray's Method to calculate the **SUH**. Note that this method results in a **SUH** with a duration of **Excess Rainfall** equal to  $T_R/4$

From Eq. 4.23  $\rightarrow WF = 0.98 \text{ mi} / (1.45 \%)^{1/2} = 0.814$

From Eq. 4.25  $\rightarrow SF = 9.27 \times (0.814)^{0.562} = 8.26$

From Eq. 4.26  $\rightarrow 8.26 \times (2.676/T_R + 0.0139) = 1 \rightarrow T_R = 24.96 \text{ min}$

From Eq. 4.24  $\rightarrow 8.26 = T_R / \gamma' \rightarrow \gamma' = 3.02$ , and

**Shape parameter,  $q = 1 + \gamma' = 4.02$**

## *SUH*: Gray's Method

- Now, we have to use Eqs. 4.28 and 4.29 to calculate the volumes of the unit hydrograph and the dimensionless hydrograph respectively, which have to be equal

$$V = 1 \text{ inch} \times 640 \text{ acre/mi}^2 \times 0.0833 \text{ ft/inch} \times 43,560 \text{ ft}^2/\text{acre} = 1,439,807.85 \text{ ft}^3$$

$$V_D = 1.43981 \times 10^6 \text{ ft}^3 = 0.25 \times 24.96 \text{ min} \times 60 \text{ sec/min} \times \Sigma cfs \rightarrow \Sigma cfs = 3845.74 \text{ ft}^3/\text{sec}$$

- The values of time ( $T$ ) are calculated knowing that there will be four points for  $T/T_R$  between 0 and 1.0 at  $0.25 \times T_R$  intervals with the exception of the first point that it will be  $0.125 \times T_R$  ( $T_R/8$ ). The following table illustrate the procedure and the calculation of the dimensionless hydrograph (Eq. 4.27) and of the **SUH** from the Gray's Method (Eq. 4.30)

The Gamma Function of  $q$  is:  $\Gamma(4.02) = 3.02 \times 2.02 \times 1.02 \times \Gamma(1.02) = 6.152992$

## *SUH*: Gray's Method

## Computation of dimensionless Hydrograph and SUH from Gray's Method ( $T$ Function)

| $T/T_R$ | $T$ (min) <sup>1)</sup> | $Q_{T/T_R}$ (%) <sup>2)</sup> | $\Sigma Q_{T/T_R}$ (%) <sup>3)</sup> | $Q_u$ (cfs) <sup>4)</sup> | $T/T_R$  | $T$ (min) <sup>1)</sup> | $Q_{T/T_R}$ (%) <sup>2)</sup> | $\Sigma Q_{T/T_R}$ (%) <sup>3)</sup> | $Q_u$ (cfs) <sup>4)</sup> |
|---------|-------------------------|-------------------------------|--------------------------------------|---------------------------|----------|-------------------------|-------------------------------|--------------------------------------|---------------------------|
| 0       | 0.00                    | 0.00                          | 0.00                                 | 0.00                      | 1.875    | 46.80                   | 8.04                          | 85.42                                | 309.21                    |
| 0.125   | 3.12                    | 0.44                          | 0.44                                 | 17.08                     | 2.125    | 53.04                   | 5.51                          | 90.93                                | 212.01                    |
| 0.375   | 9.36                    | 5.78                          | 6.22                                 | 222.11                    | 2.375    | 59.28                   | 3.62                          | 94.55                                | 139.37                    |
| 0.625   | 15.60                   | 12.71                         | 18.93                                | 488.65                    | 2.625    | 65.52                   | 2.30                          | 96.86                                | 88.58                     |
| 0.875   | 21.84                   | 16.50                         | 35.43                                | 634.63                    | 2.875    | 71.76                   | 1.42                          | 98.28                                | 54.77                     |
| 1       | 24.96                   | 16.93                         | —                                    | 651.21                    | 3.125    | 78.00                   | 0.86                          | 99.14                                | 33.10                     |
| 1.125   | 28.08                   | 16.57                         | 52.00                                | 637.16                    | 3.375    | 84.24                   | 0.51                          | 99.65                                | 19.62                     |
| 1.375   | 34.32                   | 14.27                         | 66.27                                | 548.89                    | 3.625    | 90.48                   | 0.30                          | 99.95                                | 11.44                     |
| 1.625   | 40.56                   | 11.11                         | 77.38                                | 427.16                    | $\Sigma$ |                         | 99.95                         |                                      | 3843.79                   |

1) Values in first column ( $T/T_R$ ) multiplied by  $T_R = 24.96$  min

2) Ordinates of dimensionless hydrograph from Eq. 4.27. The value for  $T_R = 16.93\%$  is included only for the purpose of computing the peak discharge for the  $SUH = 651.21$  ft<sup>3</sup>/sec

3) Accumulated dimensionless hydrograph. It approaches 100%

4) Ordinates of the **SUH** from Eq. 4.30, values of  $Q_{T/T_R}$  multiplied by  $\Sigma cfs = 3845.74$  ft<sup>3</sup>/sec from Eq. 4.29. Note its match with the sum of values in the Table ( $Q_u$  peak at  $T_R$  is excluded from sum)

Integration of the **SUH** over the watershed will yield the **Excess Rainfall**

Time interval =  $T_R/4 = 6.24$  min  
(or  $9.36 - 3.12$ , or  $3.12 \times 2$ )

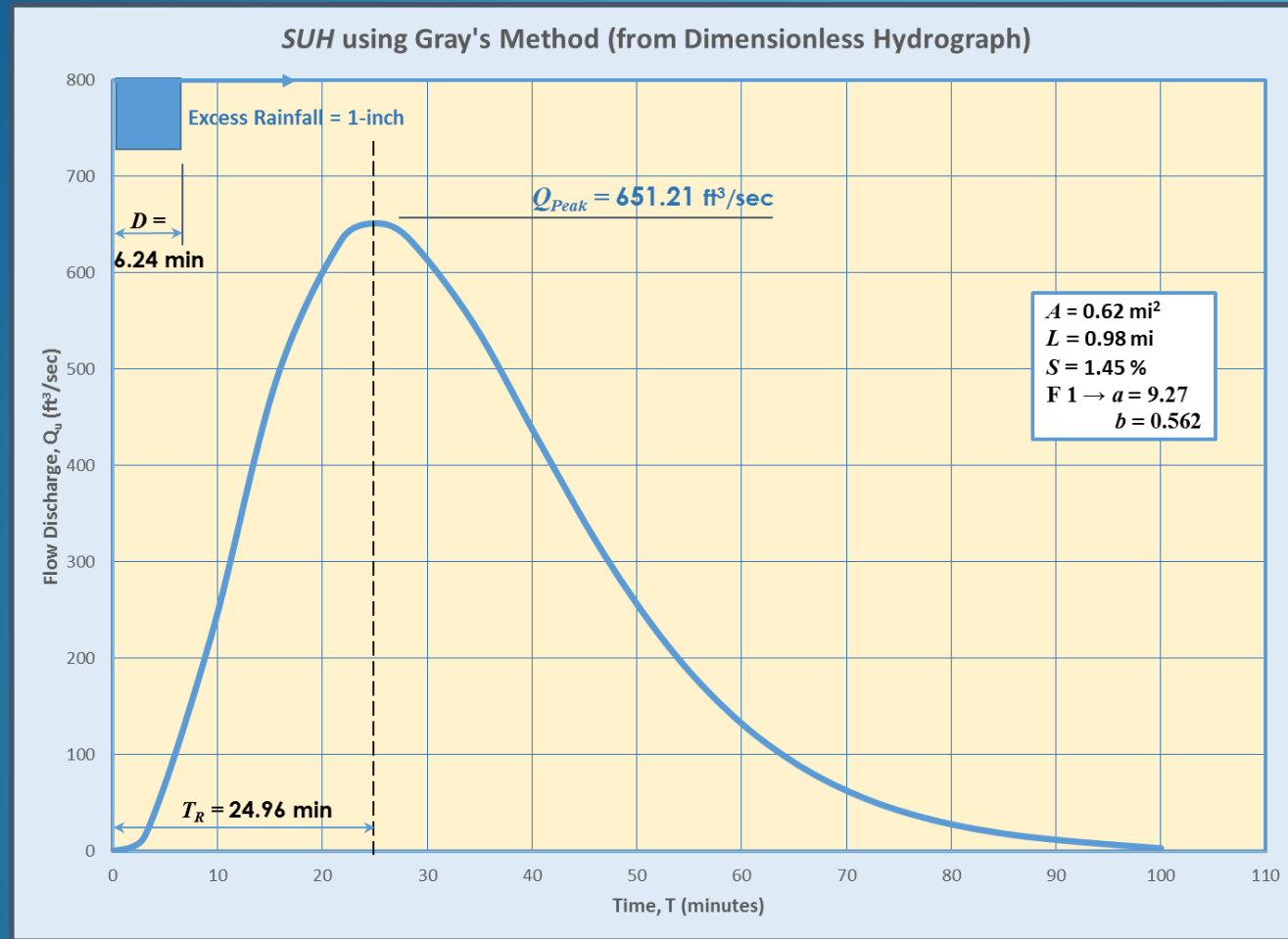
$V_D = 3843.79$  cfs  $\times 6.24$  min  $\times 60 = 1,439,115$  ft<sup>3</sup>

$A = 0.62$  mi<sup>2</sup> =  $2.788 \times 10^7$  ft<sup>2</sup>/mi<sup>2</sup> = 17,285,600 ft<sup>2</sup>

$V_D/A = 0.08326$  ft = 1.0 inch

This proves that the **SUH** is correct. As expected **Excess Rainfall** is **1.0 inch**

# SUH: Gray's Method



**SUH: Gray's Method**

# Florida Department of Environmental Protection



## Questions?

## Thank you!

